CS462 Image Processing

Chapter 7

Discrete Cosine Transform

and

JPEG Compression

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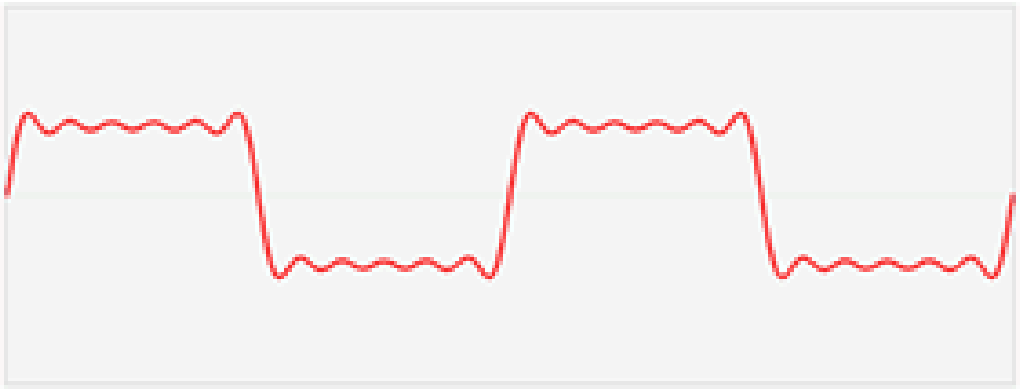
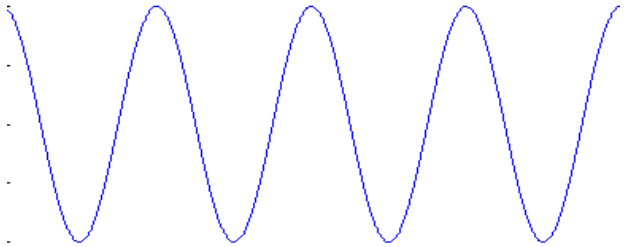
Fourier transform

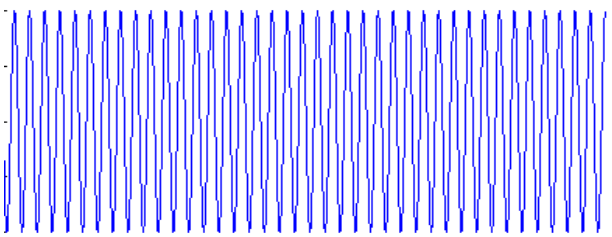
Jean-Baptiste Joseph Fourier

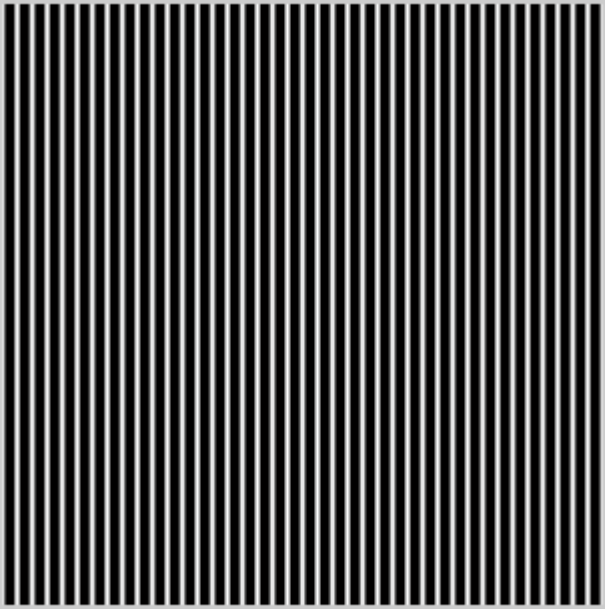
The **Fourier transform** decomposes a function of time (a *signal*) into the frequencies that make it up



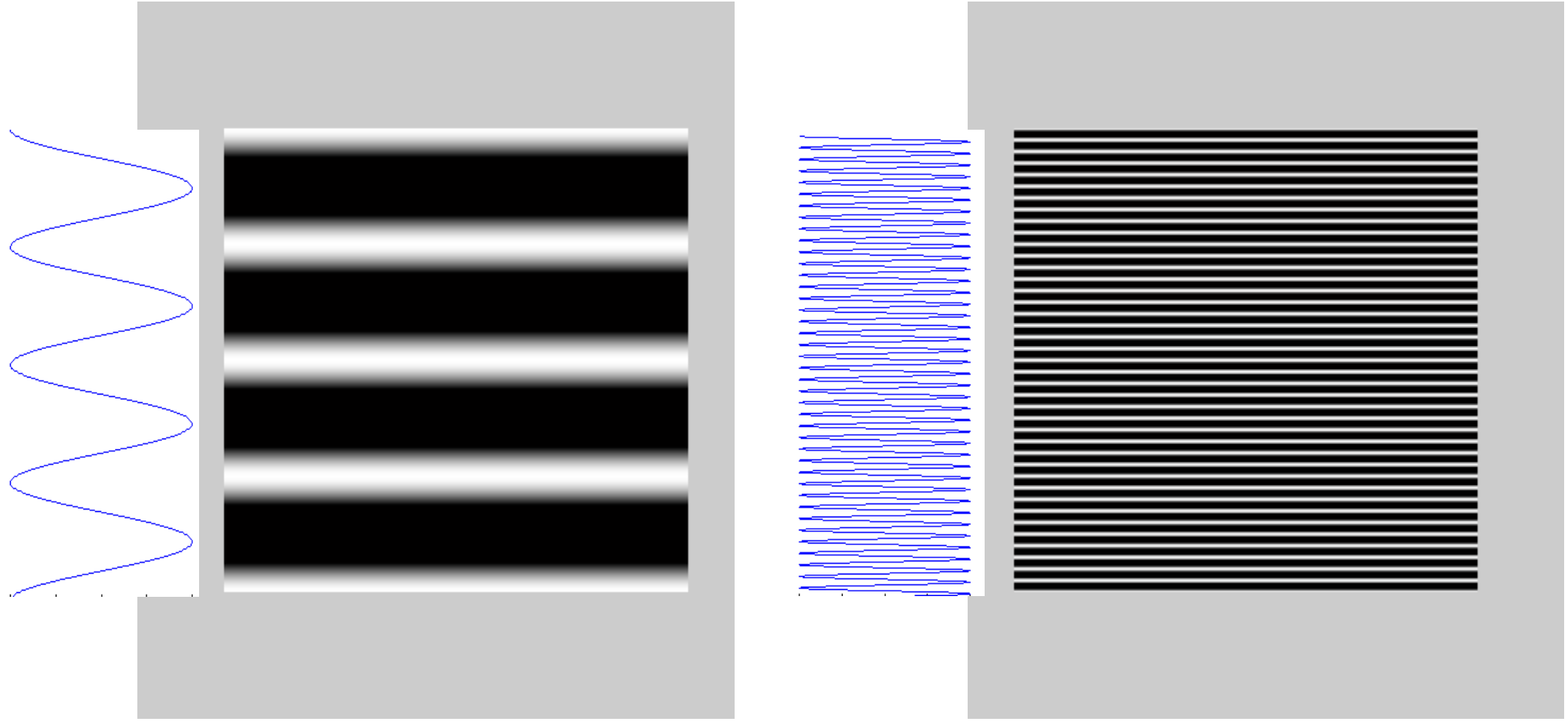
Frequency



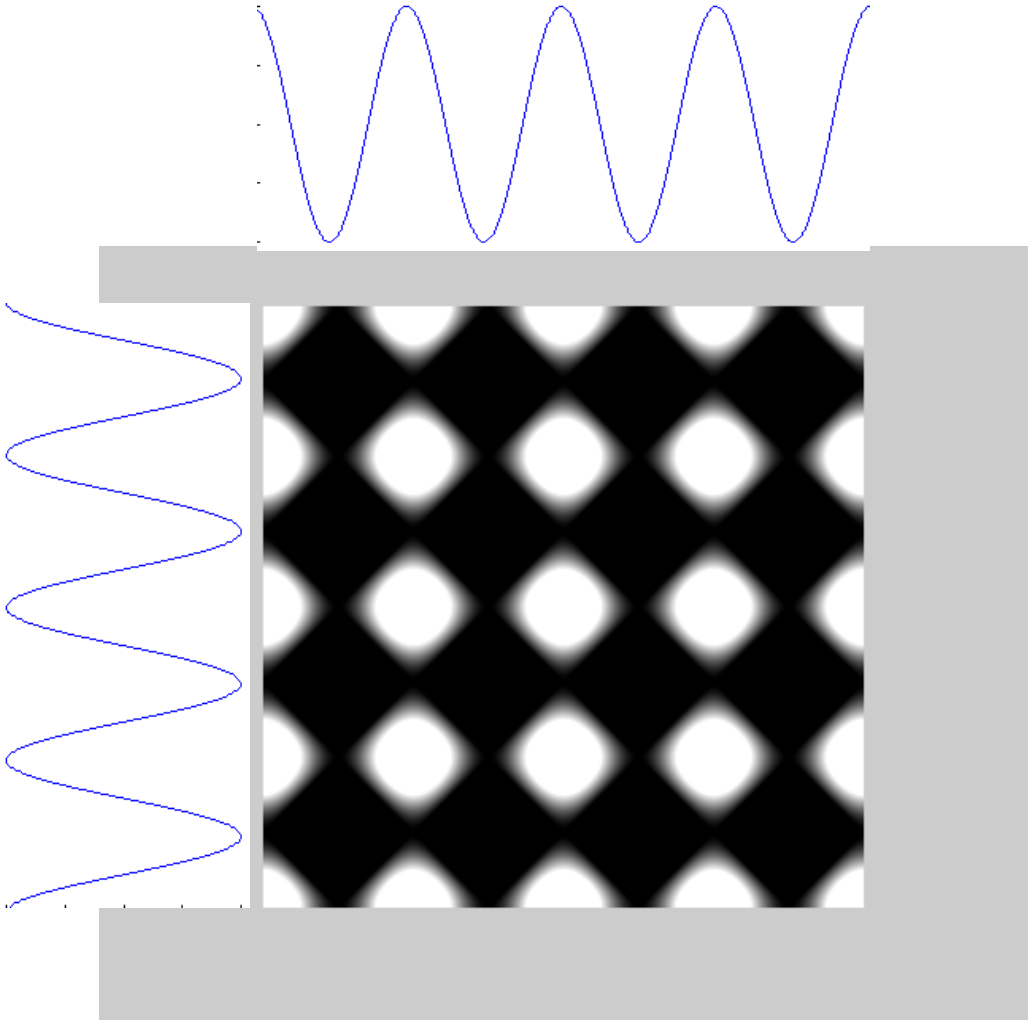


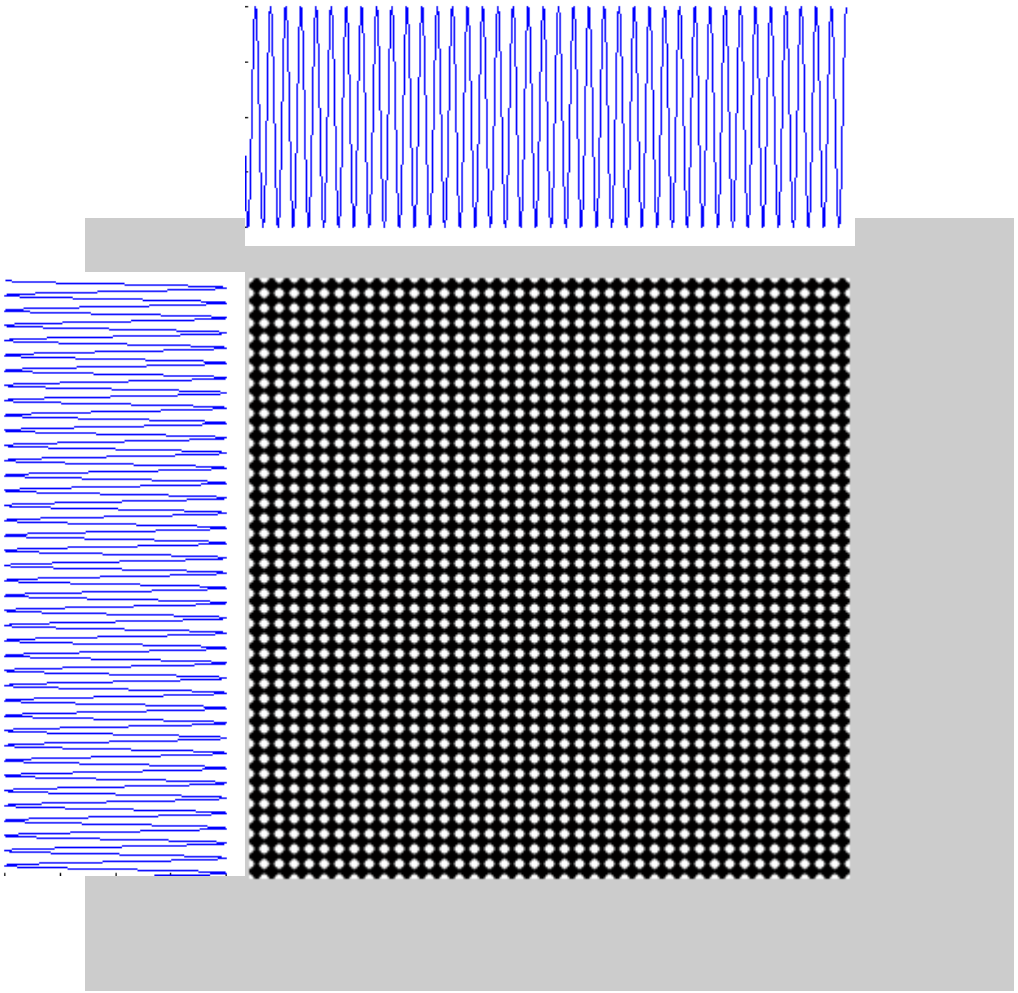


Frequency

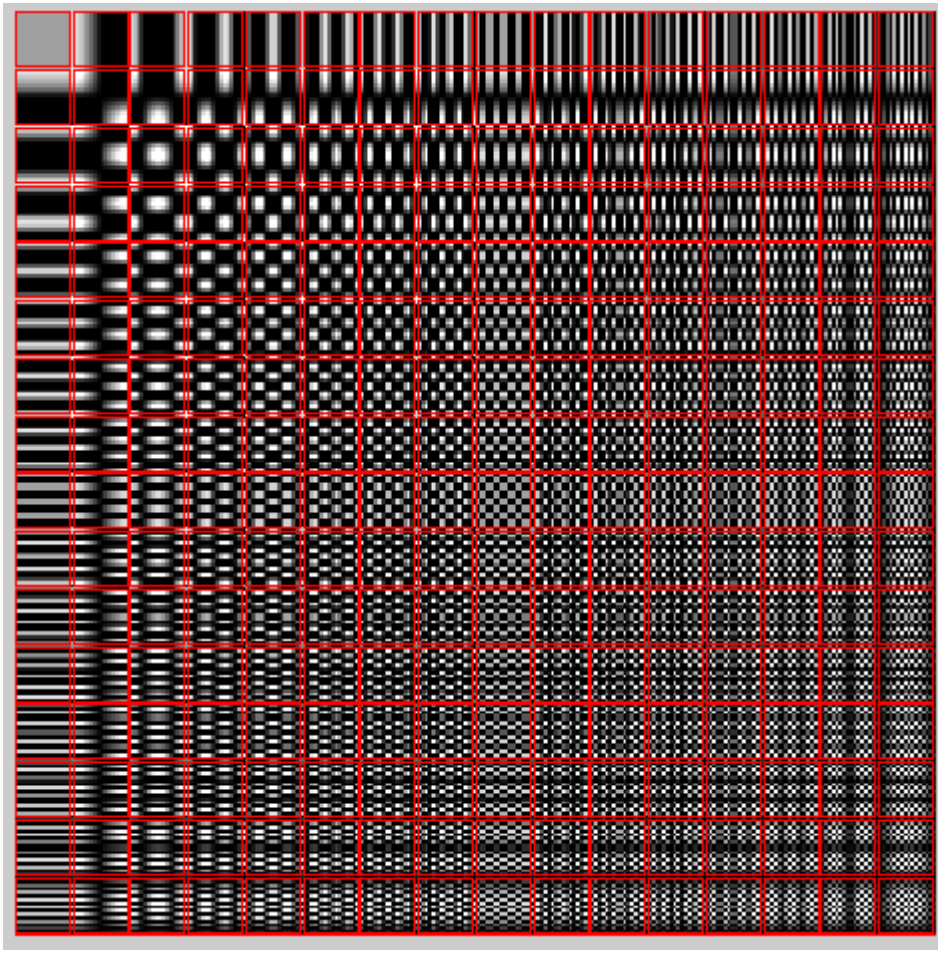


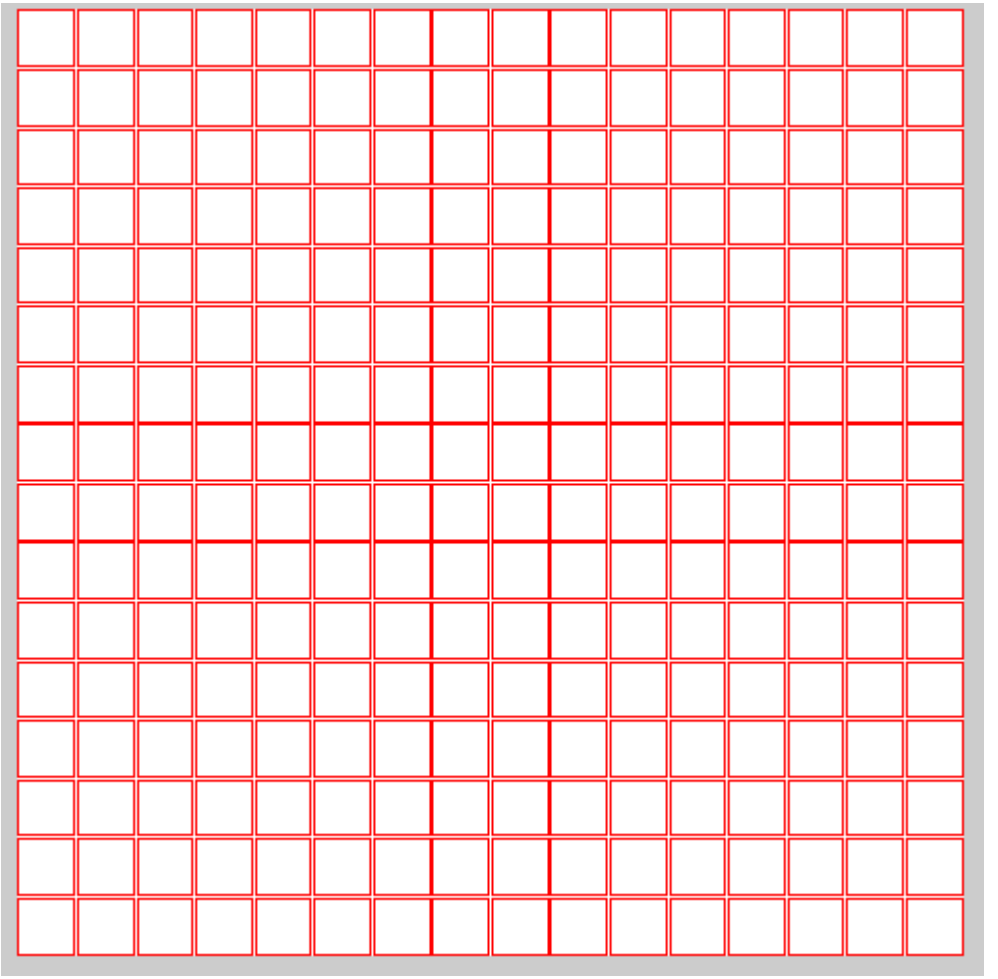
Frequency





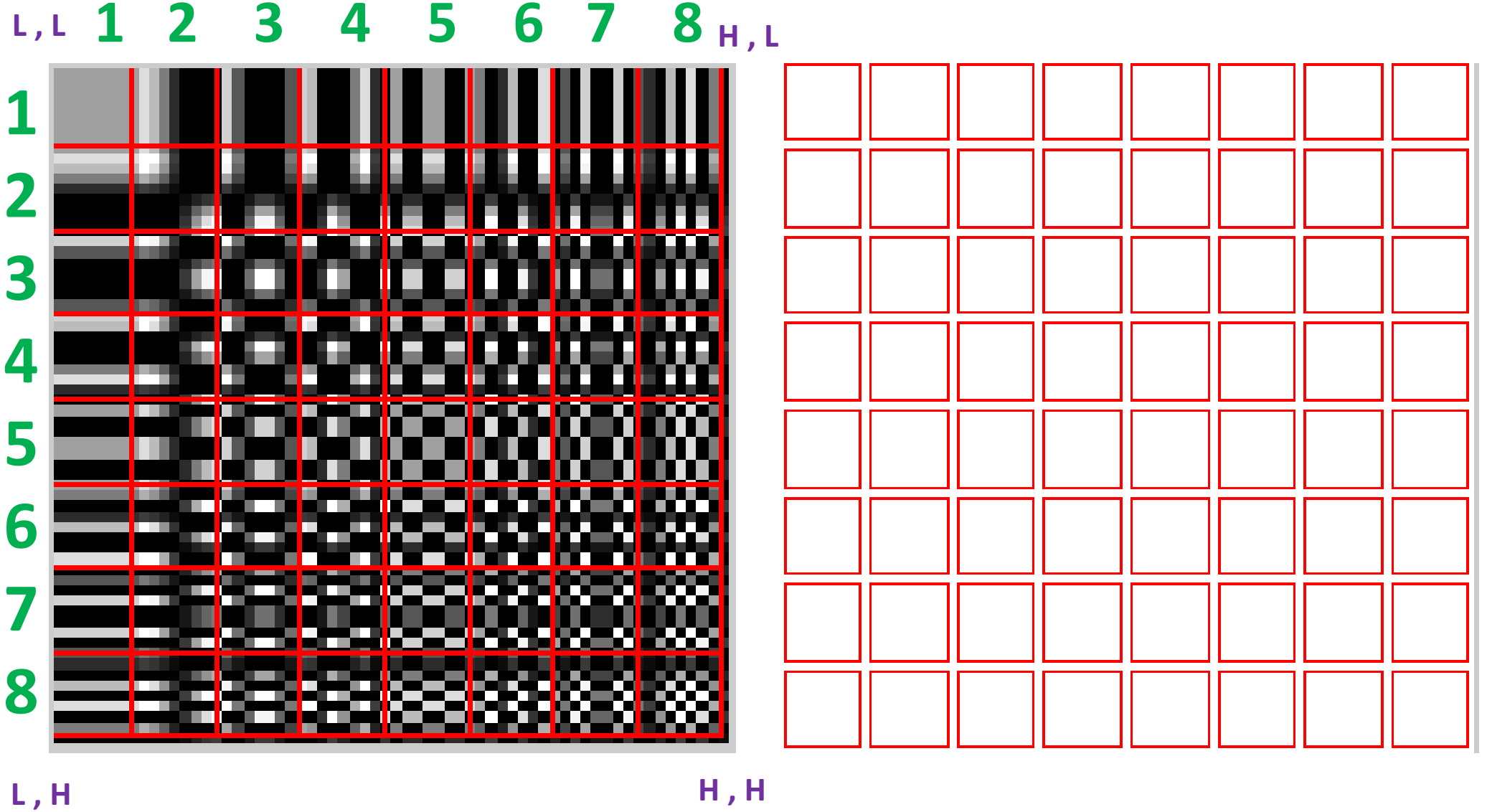
Sum of cosine functions oscillating at different frequencies





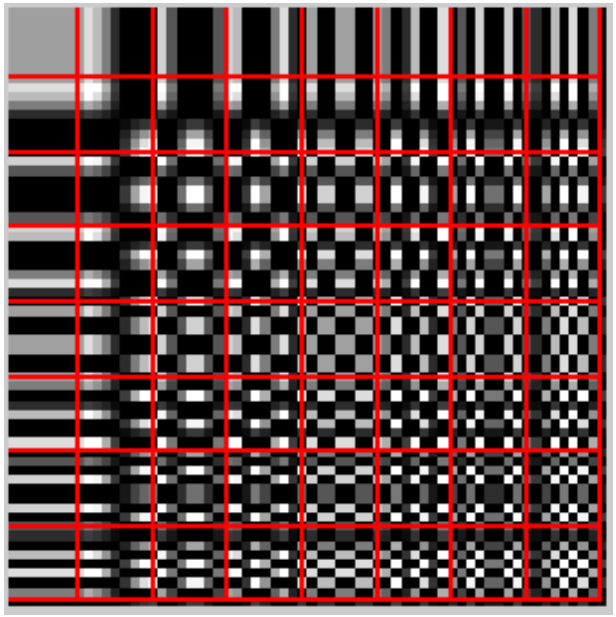
Discrete Cosine Basis Function

Sum of cosine functions oscillating at different frequencies

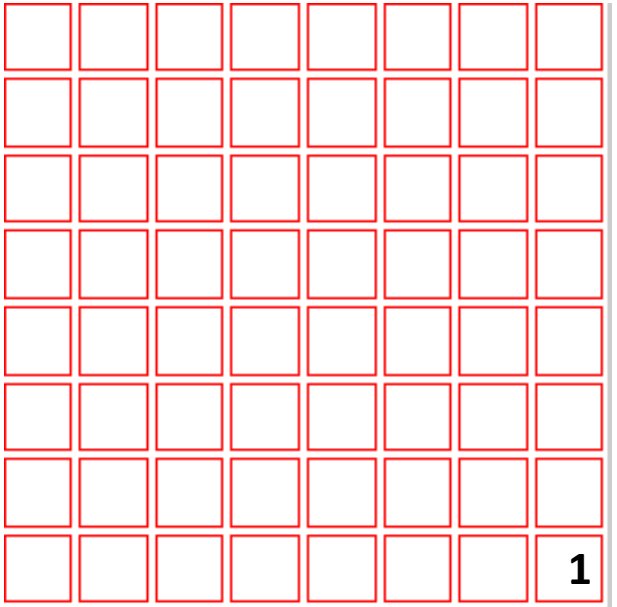


Discrete Cosine Basis Function

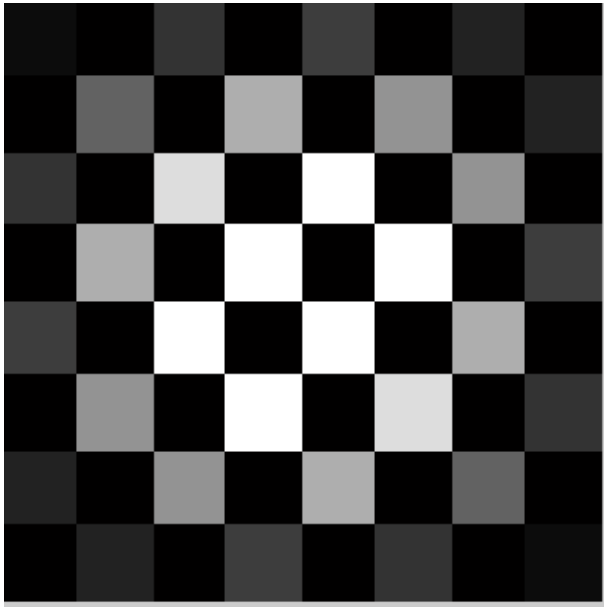
Sum of cosine functions oscillating at different frequencies



Discrete Cosine Basis Function

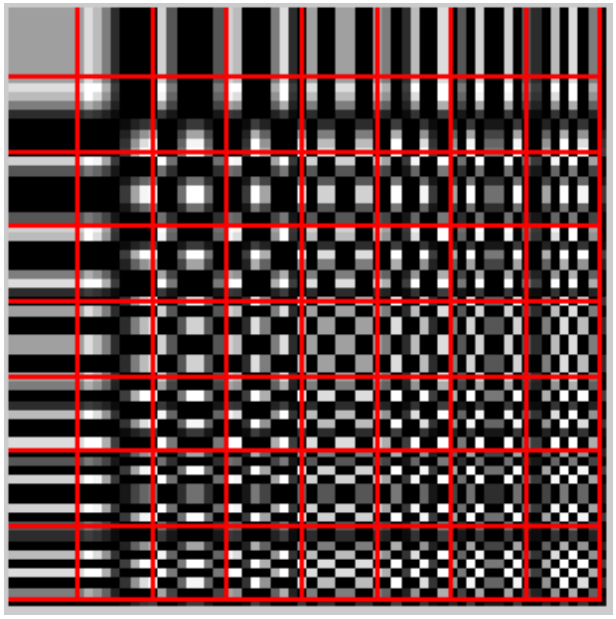


Discrete Cosine Coefficient

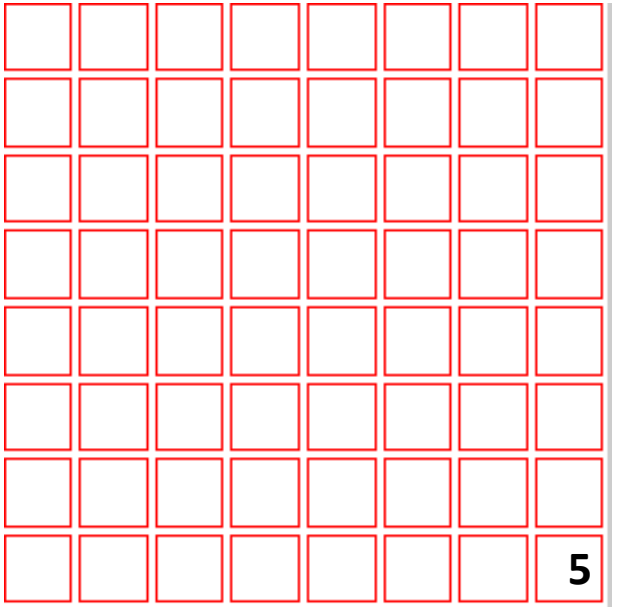


Sum of cosine functions

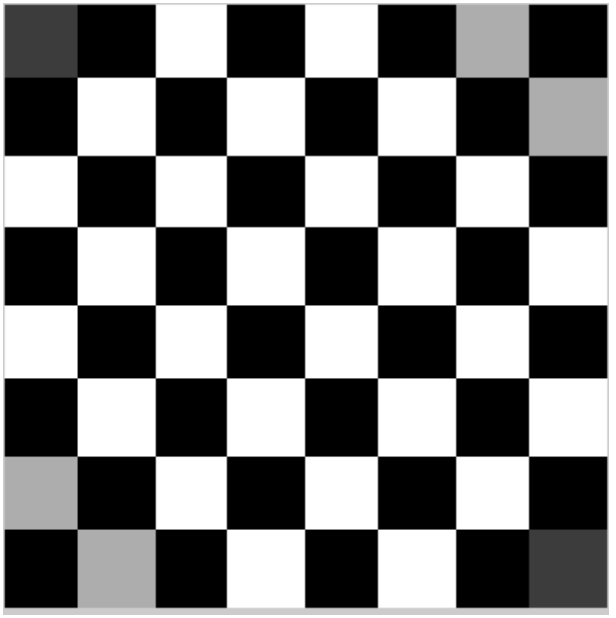
Sum of cosine functions oscillating at different frequencies



Discrete Cosine Basis Function

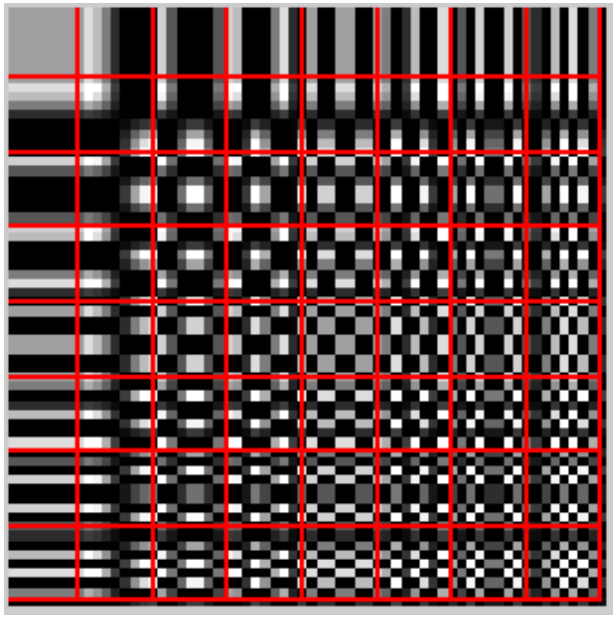


Discrete Cosine Coefficient

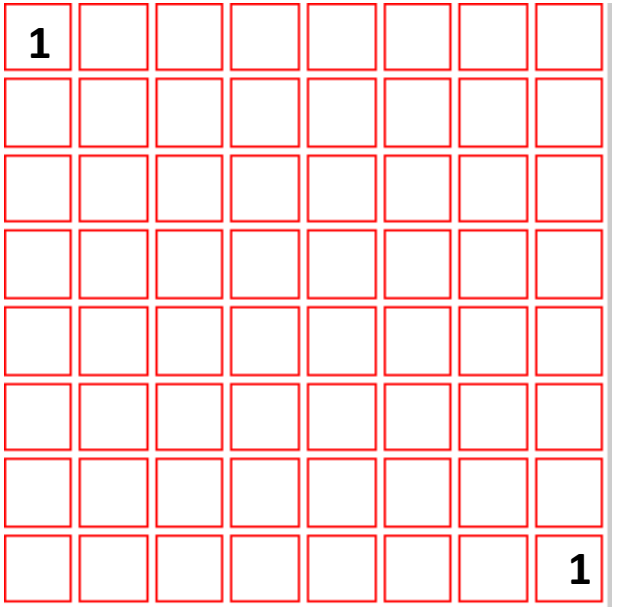


Sum of cosine functions

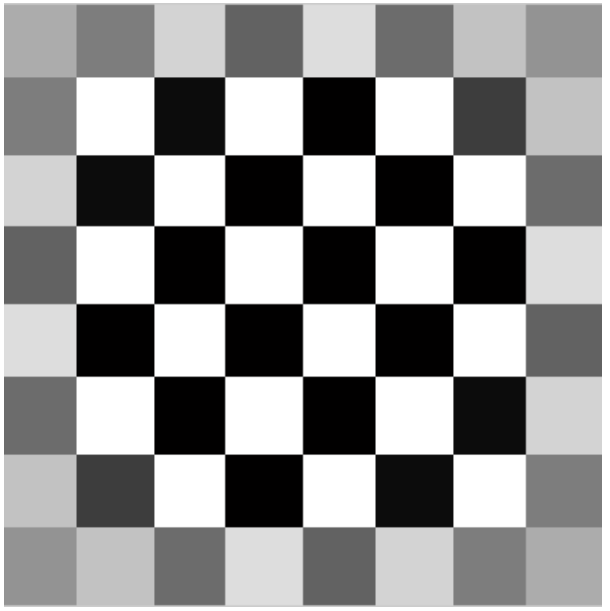
Sum of cosine functions oscillating at different frequencies



Discrete Cosine Basis Function

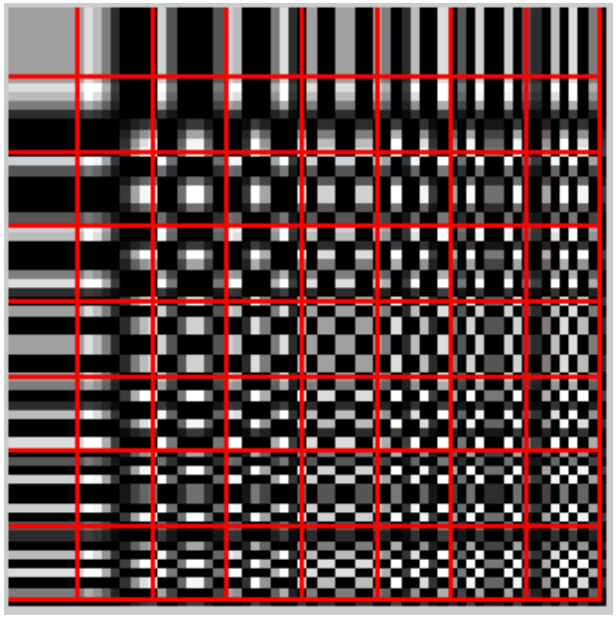


Discrete Cosine Coefficient

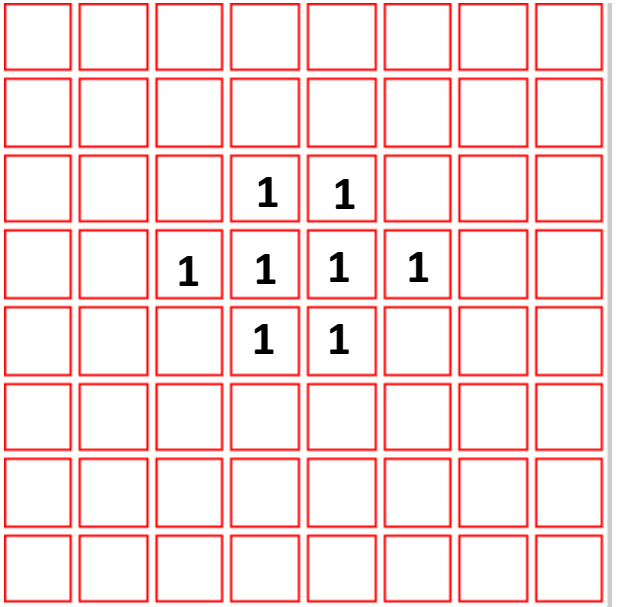


Sum of cosine functions

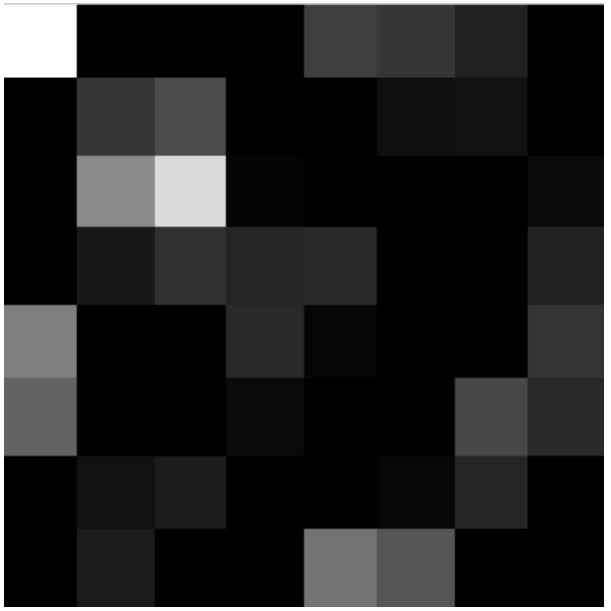
Sum of cosine functions oscillating at different frequencies



Discrete Cosine Basis Function



Discrete Cosine Coefficient



Sum of cosine functions

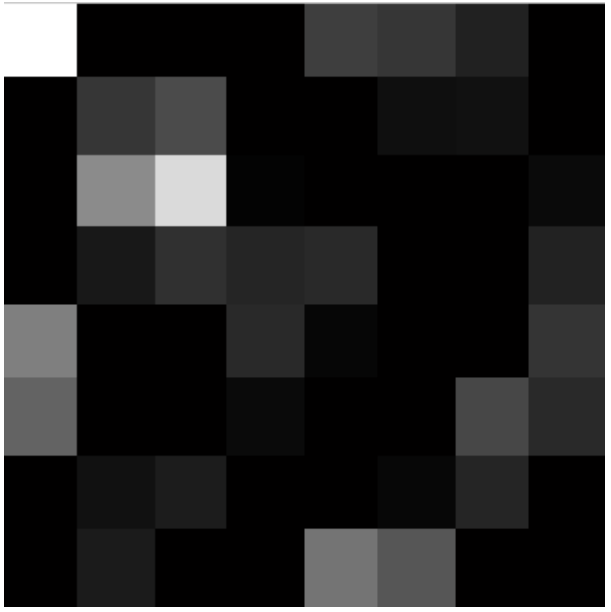
8 pixels

8 x 8 pixels

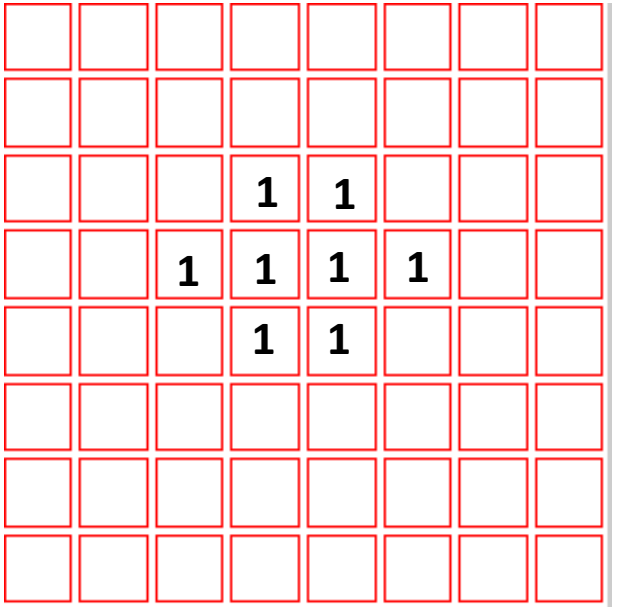
Discrete Cosine Transform

A discrete cosine transform (DCT) expresses a finite sequence of data points in terms of a sum of cosine functions oscillating at different frequencies

Nasir Ahmed



DCT



Sum of cosine functions

Discrete Cosine Coefficient

Discrete Cosine Transform

Forward DCT Formula

$$\mathrm{B(r,c)} = \alpha_r \alpha_c \sum_{m=0}^{R-1} \sum_{n=0}^{C-1} I(m,n) cos \frac{\pi(2m+1)r}{2R} cos \frac{\pi(2n+1)c}{2N}$$

$$\alpha_r = \begin{cases} \frac{1}{\sqrt{R}} & ; r = 0 \\ \sqrt{\frac{2}{R}} & ; \ 1 \le r \le R-1 \end{cases}$$

$$\alpha_c = \begin{cases} \frac{1}{\sqrt{C}} & ; \ c = 0 \\ \sqrt{\frac{2}{C}} & ; \ 1 \le r \le C-1 \end{cases}$$

I is Image
B is Transform image
r is row
R is image height
c is column
C is image width

Discrete Cosine Transform

$$\mathrm{B(r,c)} = \alpha_r \alpha_c \sum_{m=0}^{R-1} \sum_{n=0}^{C-1} I(m,n) cos \frac{\pi(2m+1)r}{2R} cos \frac{\pi(2n+1)c}{2N}$$

$$\alpha_r = \begin{cases} \frac{1}{\sqrt{R}} & ; r = 0 \\ \sqrt{\frac{2}{R}} & ; \ 1 \le r \le R-1 \end{cases}$$

$$\alpha_c = \begin{cases} \frac{1}{\sqrt{C}} & ; \ c = 0 \\ \sqrt{\frac{2}{C}} & ; \ 1 \le r \le C-1 \end{cases}$$

1	2
3	4

I

I is Image
B is Transform image
r is row
R is image height
c is column
C is image width

Discrete Cosine Transform

$$B(r,c) = \alpha_r \alpha_c \sum_{m=0}^{R-1} \sum_{n=0}^{C-1} I(m,n) \cos\frac{\pi(2m+1)r}{2R} \cos\frac{\pi(2n+1)c}{2N}$$

$$\alpha_r = \begin{cases} \frac{1}{\sqrt{R}} & ; r = 0 \\ \sqrt{\frac{2}{R}} & ; \ 1 \le r \le R-1 \end{cases}$$

$$\alpha_c = \begin{cases} \frac{1}{\sqrt{C}} & ; \ c = 0 \\ \sqrt{\frac{2}{C}} & ; \ 1 \le r \le C-1 \end{cases}$$

1	2
3	4

I

I is Image
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Discrete Cosine Transform

Forward DCT Formular

$$\alpha_r = \frac{1}{\sqrt{R}} = \frac{1}{\sqrt{2}} = 0.7071$$

$$\alpha_c = \frac{1}{\sqrt{C}} = \frac{1}{\sqrt{2}} = 0.7071$$

$$\mathrm{B(r,c)} = \alpha_r \alpha_c \sum_{m=0}^{R-1} \sum_{n=0}^{C-1} I(m,n) cos \frac{\pi(2m+1)r}{2R} + cos \frac{\pi(2n+1)c}{2N}$$

1	2
3	4

I

B(0,0)

$$= 0.7071 \times 0.7071$$

$$\times \left(1cos\left(\frac{\pi(2\times 0+1)0}{2\times 2}\right) cos\left(\frac{\pi(2\times 0+1)0}{2\times 2}\right) + 2cos\left(\frac{\pi(2\times 0+1)0}{2\times 2}\right) cos\left(\frac{\pi(2\times 1+1)0}{2\times 2}\right) + 3cos\left(\frac{\pi(2\times 1+1)0}{2\times 2}\right) cos\left(\frac{\pi(2\times 0+1)0}{2\times 2}\right) \right.$$

5	

B

Discrete Cosine Transform

$$B(r,c) = \alpha_r \alpha_c \sum_{m=0}^{R-1} \sum_{n=0}^{C-1} I(m,n) \cos\frac{\pi(2m+1)r}{2R} \cos\frac{\pi(2n+1)c}{2N}$$

$$\alpha_r = \begin{cases} \frac{1}{\sqrt{R}} & ; r = 0 \\ \sqrt{\frac{2}{R}} & ; 1 \le r \le R-1 \end{cases}$$

$$\alpha_c = \begin{cases} \frac{1}{\sqrt{C}} & ; c = 0 \\ \sqrt{\frac{2}{C}} & ; 1 \le r \le C-1 \end{cases}$$

1	2
3	4

I

I is Image
B is Transform image
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R is image height
c is column
C is image width

Discrete Cosine Transform

$$\alpha_r = \frac{1}{\sqrt{R}} = \frac{1}{\sqrt{2}} = 0.7071$$

$$\alpha_c = \sqrt{\frac{2}{C}} = \sqrt{\frac{2}{2}} = 1$$

1	2
3	4

I

B(0,1)

$$= 0.7071 \times 1$$

$$\times \left(\; 1cos\left(\frac{\pi(2\times 0+1)0}{2\times 2}\right) cos\left(\frac{\pi(2\times 0+1)1}{2\times 2}\right) + 2cos\left(\frac{\pi(2\times 0+1)0}{2\times 2}\right) cos\left(\frac{\pi(2\times 1+1)1}{2\times 2}\right)\right.$$

5	-1

B

Discrete Cosine Transform

$$B(r,c) = \alpha_r \alpha_c \sum_{m=0}^{R-1} \sum_{n=0}^{C-1} I(m,n) \cos\frac{\pi(2m+1)r}{2R} \cos\frac{\pi(2n+1)c}{2N}$$

$$\alpha_r = \begin{cases} \frac{1}{\sqrt{R}} & ; r = 0 \\ \sqrt{\frac{2}{R}} & ; \ 1 \le r \le R-1 \end{cases}$$

$$\alpha_c = \begin{cases} \frac{1}{\sqrt{C}} & ; \ c = 0 \\ \sqrt{\frac{2}{C}} & ; \ 1 \le r \le C-1 \end{cases}$$

1	2
3	4

I

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Discrete Cosine Transform

$$\alpha_r = \sqrt{\frac{2}{R}} = \sqrt{\frac{2}{2}} = 1$$

$$\alpha_c = \frac{1}{\sqrt{C}} = \frac{1}{\sqrt{2}} = 0.7071$$

1	2
3	4

I

B(1,0)

$$= 1 \times 0.707$$

$$\times \left(\quad 1cos\left(\frac{\pi(2\times 0+1)1}{2\times 2}\right) cos\left(\frac{\pi(2\times 0+1)0}{2\times 2}\right) + 2cos\left(\frac{\pi(2\times 0+1)1}{2\times 2}\right) cos\left(\frac{\pi(2\times 1+1)0}{2\times 2}\right)\right.$$

5	-1
-2	

B

Discrete Cosine Transform

$$B(r,c) = \alpha_r \alpha_c \sum_{m=0}^{R-1} \sum_{n=0}^{C-1} I(m,n) \cos\frac{\pi(2m+1)r}{2R} \cos\frac{\pi(2n+1)c}{2N}$$

$$\alpha_r = \begin{cases} \frac{1}{\sqrt{R}} & ; r = 0 \\ \sqrt{\frac{2}{R}} & ; \ 1 \le r \le R-1 \end{cases}$$

$$\alpha_c = \begin{cases} \frac{1}{\sqrt{C}} & ; \ c = 0 \\ \sqrt{\frac{2}{C}} & ; \ 1 \le r \le C-1 \end{cases}$$

1	2
3	4

I

I is Image
B is Transform image
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Discrete Cosine Transform

$$\alpha_r = \sqrt{\frac{2}{R}} = \sqrt{\frac{2}{2}} = 1$$

$$\alpha_c = \sqrt{\frac{2}{C}} = \sqrt{\frac{2}{2}} = 1$$

1	2
3	4

I

B(1,1)

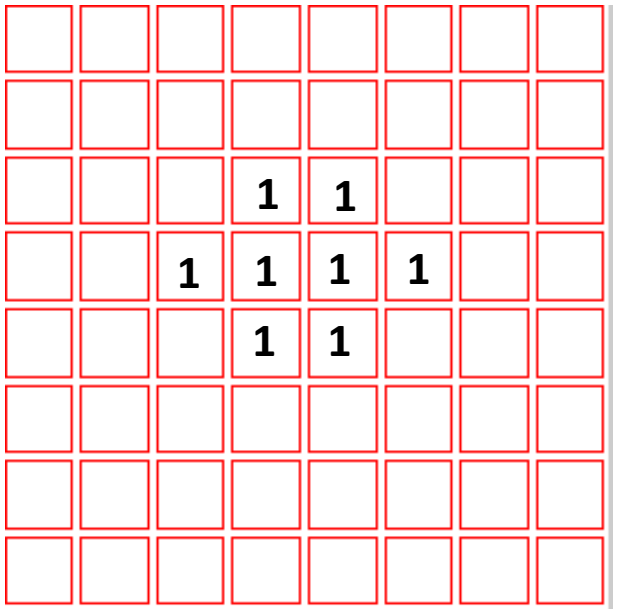
$$= 1 \times 1$$

$$\times \left(\; 1cos\left(\frac{\pi(2\times 0+1)1}{2\times 2}\right) cos\left(\frac{\pi(2\times 0+1)1}{2\times 2}\right) + 2cos\left(\frac{\pi(2\times 0+1)1}{2\times 2}\right) cos\left(\frac{\pi(2\times 1+1)1}{2\times 2}\right)\right.$$

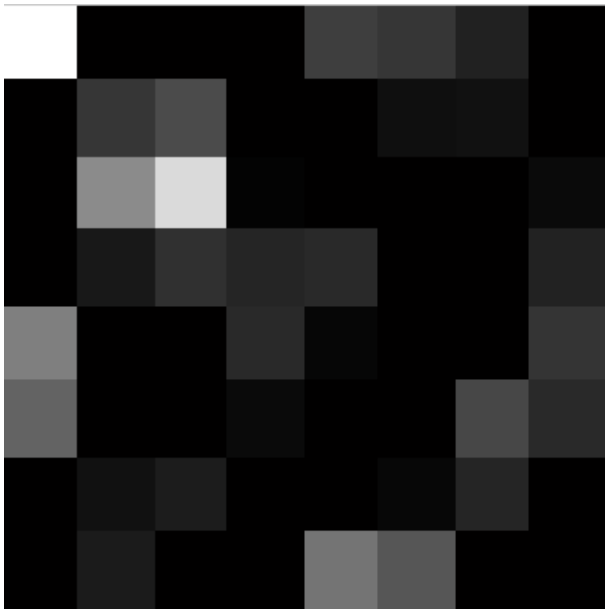
5	-1
-2	0

B

Inverse Discrete Cosine Transform



IDCT



Discrete Cosine Coefficient

Sum of cosine functions

Inverse Discrete Cosine Transform

Inverse Discrete Cosine Transform Formula

$$I(m,n) = \sum_{r=0}^{M-1} \sum_{c=0}^{N-1} \alpha_r \alpha_c B(r,c) cos\left(\frac{\pi(2m+1)r}{2M}\right) cos\left(\frac{\pi(2n+1)c}{2N}\right)$$

$$\alpha_r = \begin{cases} \frac{1}{\sqrt{M}} & ;\ r = 0 \\ \sqrt{\frac{2}{M}} & ;\ 1 \le r \le M-1 \end{cases}$$

$$\alpha_c = \begin{cases} \frac{1}{\sqrt{N}} & ;\ c = 0 \\ \sqrt{\frac{2}{N}} & ;\ 1 \le c \le N-1 \end{cases}$$

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Inverse Discrete Cosine Transform

Inverse Discrete Cosine Transform Formula

$$I(m,n) = \sum_{r=0}^{M-1} \sum_{c=0}^{N-1} \alpha_r \alpha_c B(r,c) cos\left(\frac{\pi(2m+1)r}{2M}\right) cos\left(\frac{\pi(2n+1)c}{2N}\right)$$

5	-1
-2	0

B

$$\alpha_r = \begin{cases} \frac{1}{\sqrt{M}} & ; r = 0 \\ \sqrt{\frac{2}{M}} & ; 1 \le r \le M-1 \end{cases}$$

$$\alpha_c = \begin{cases} \frac{1}{\sqrt{N}} & ; c = 0 \\ \sqrt{\frac{2}{N}} & ; 1 \le c \le N-1 \end{cases}$$

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R is image height
c is column
C is image width

Inverse Discrete Cosine Transform

Precompute

5	-1
-2	0

B

$$\alpha_r = \begin{cases} \frac{1}{\sqrt{M}} = \frac{1}{\sqrt{2}} = 0.7071 & ; r = 0 \\ \sqrt{\frac{2}{M}} = \sqrt{\frac{2}{2}} = 1 ; & 1 \le r \le M-1 \end{cases}$$

$$\alpha_c = \begin{cases} \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{2}} = 0.7071 & ; c = 0 \\ \sqrt{\frac{2}{N}} = \sqrt{\frac{2}{2}} = 1 ; & 1 \le c \le N-1 \end{cases}$$

Inverse Discrete Cosine Transform

$$I(m,n) = \sum_{r=0}^{M-1} \sum_{c=0}^{N-1} \alpha_r \alpha_c B(r,c) cos\left(\frac{\pi(2m+1)}{2M}\right) r \, cos\left(\frac{\pi(2n+1)}{2N}\right) c$$

5	-1
-2	0

B

$$I(0,0)$$

$$= \left(0.7071 \times 0.7071 \times 5 \times cos\left(\frac{\pi(2\times 0+1)0}{2\times 2}\right) \times cos\left(\frac{\pi(2\times 0+1)0}{2\times 2}\right)\right)$$

$$+ \left(0.7071 \times 1 \times -1 \times cos\left(\frac{\pi(2\times 0+1)0}{2\times 2}\right) \times cos\left(\frac{\pi(2\times 0+1)1}{2\times 2}\right)\right)$$

$$+ \left(1 \times 0.707 \times -2 \times cos\left(\frac{\pi(2\times 0+1)1}{2\times 2}\right) \times cos\left(\frac{\pi(2\times 0+1)0}{2\times 2}\right)\right)$$

$$+ \left(1 \times 1 \times 0 \times cos\left(\frac{\pi(2\times 0+1)1}{2\times 2}\right) \times cos\left(\frac{\pi(2\times 0+1)1}{2\times 2}\right)\right)$$

$$I(0,0)$$
$$= (0.7071 \times 0.7071 \times 5 \times 1 \times 1) + (0.7071 \times 1 \times -1 \times 1 \times 0.7071) + (1 \times 0.707 \times -2 \times 0.7071 \times 1)$$
$$+ (1 \times 1 \times 0 \times 0.7071 \times 0.7071)$$

$$I(0,0) = (2.5) + (-0.5) + (-1) + (0) = 1$$

1	

I

Inverse Discrete Cosine Transform

$$I(0,1)$$
$$= \left(0.7071 \times 0.7071 \times 5 \times cos\left(\frac{\pi(2 \times 0 + 1)0}{2 \times 2}\right) \times cos\left(\frac{\pi(2 \times 1 + 1)0}{2 \times 2}\right)\right)$$
$$+ \left(0.7071 \times 1 \times -1 \times cos\left(\frac{\pi(2 \times 0 + 1)0}{2 \times 2}\right) \times cos\left(\frac{\pi(2 \times 1 + 1)1}{2 \times 2}\right)\right)$$
$$+ \left(1 \times 0.707 \times -2 \times cos\left(\frac{\pi(2 \times 0 + 1)1}{2 \times 2}\right) \times cos\left(\frac{\pi(2 \times 1 + 1)0}{2 \times 2}\right)\right)$$
$$+ \left(1 \times 1 \times 0 \times cos\left(\frac{\pi(2 \times 0 + 1)1}{2 \times 2}\right) \times cos\left(\frac{\pi(2 \times 1 + 1)1}{2 \times 2}\right)\right)$$

$$I(0,1)$$
$$= (0.7071 \times 0.7071 \times 5 \times 1 \times 1) + (0.7071 \times 1 \times -1 \times 1 \times -0.7071)$$
$$+ (1 \times 0.707 \times -2 \times 0.7071 \times 1) + (1 \times 1 \times 0 \times 0.7071 \times -0.7071)$$

$$I(0,1) = (2.5) + (0.5) + (-1) + (0) = 2$$

5	-1
-2	0

B

1	2

I

Inverse Discrete Cosine Transform

$$I(1,0)$$

$$= \left(0.7071 \times 0.7071 \times 5 \times cos\left(\frac{\pi(2 \times 1 + 1)0}{2 \times 2}\right) \times cos\left(\frac{\pi(2 \times 0 + 1)0}{2 \times 2}\right)\right)$$

$$+ \left(0.7071 \times 1 \times -1 \times cos\left(\frac{\pi(2 \times 1 + 1)0}{2 \times 2}\right) \times cos\left(\frac{\pi(2 \times 0 + 1)1}{2 \times 2}\right)\right)$$

$$+ \left(1 \times 0.707 \times -2 \times cos\left(\frac{\pi(2 \times 1 + 1)1}{2 \times 2}\right) \times cos\left(\frac{\pi(2 \times 0 + 1)0}{2 \times 2}\right)\right)$$

$$+ \left(1 \times 1 \times 0 \times cos\left(\frac{\pi(2 \times 1 + 1)1}{2 \times 2}\right) \times cos\left(\frac{\pi(2 \times 0 + 1)1}{2 \times 2}\right)\right)$$

$$I(1,0)$$
$$= (0.7071 \times 0.7071 \times 5 \times 1 \times 1) + (0.7071 \times 1 \times -1 \times 1 \times 0.707)$$
$$+ (1 \times 0.707 \times -2 \times -0.7071 \times 1) + (1 \times 1 \times 0 \times -0.7071 \times 0.7071)$$

$$I(1,0) = (2.5) + (-0.5) + (1) + (0) = 3$$

5	-1
-2	0

B

1	2
3	

I

Inverse Discrete Cosine Transform

$$I(1,1)$$

$$= \left(0.7071 \times 0.7071 \times 5 \times cos\left(\frac{\pi(2\times 1+1)0}{2\times 2}\right) \times cos\left(\frac{\pi(2\times 1+1)0}{2\times 2}\right)\right)$$

$$+ \left(0.7071 \times 1 \times -1 \times cos\left(\frac{\pi(2\times 1+1)0}{2\times 2}\right) \times cos\left(\frac{\pi(2\times 1+1)1}{2\times 2}\right)\right)$$

$$+ \left(1 \times 0.707 \times -2 \times cos\left(\frac{\pi(2\times 1+1)1}{2\times 2}\right) \times cos\left(\frac{\pi(2\times 1+1)0}{2\times 2}\right)\right)$$

$$+ \left(1 \times 1 \times 0 \times cos\left(\frac{\pi(2\times 1+1)1}{2\times 2}\right) \times cos\left(\frac{\pi(2\times 1+1)1}{2\times 2}\right)\right)$$

$$I(1,1)$$
$$= (0.7071 \times 0.7071 \times 5 \times 1 \times 1) + (0.7071 \times 1 \times -1 \times 1 \times -0.707)$$
$$+ (1 \times 0.707 \times -2 \times -0.7071 \times 1) + (1 \times 1 \times 0 \times -0.7071 \times -0.7071)$$

$$I(1,1) = (2.5) + (0.5) + (1) + (0) = 4$$

5	-1
-2	0

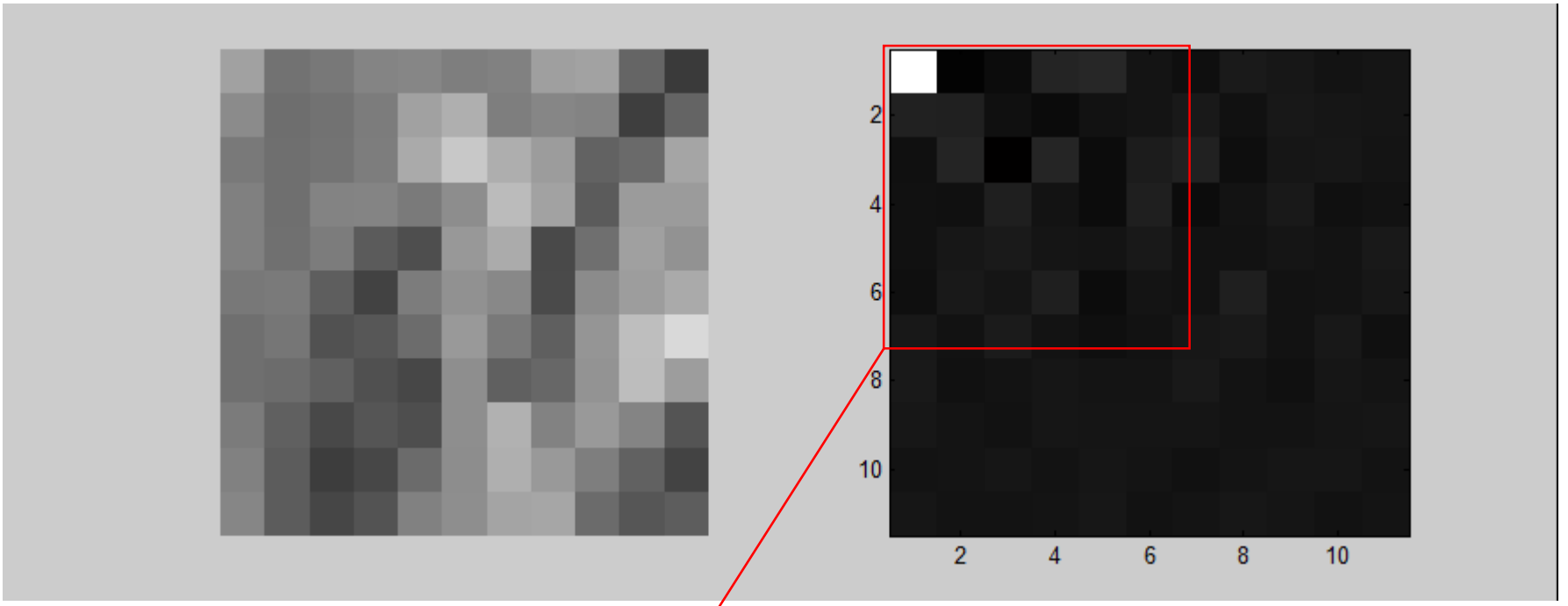
B

1	2
3	4

I

Relationship between image size and DCT coefficients

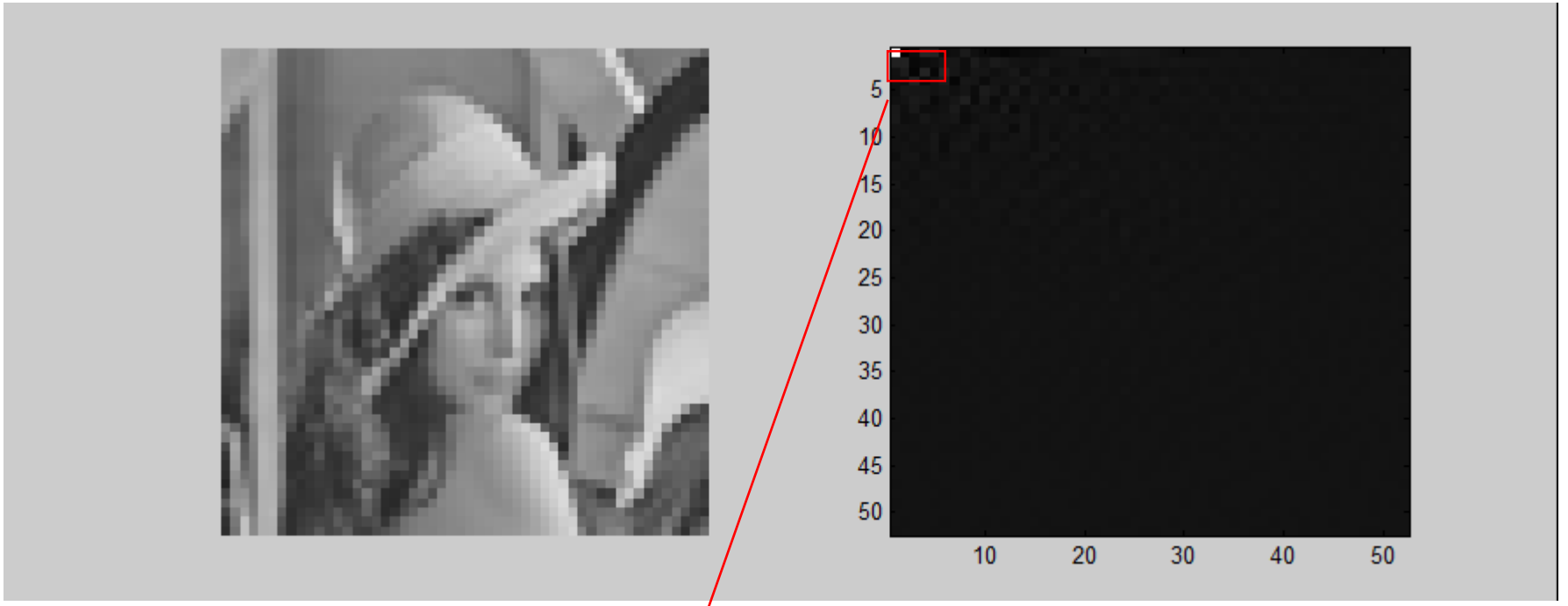
11 x 11



	1	2	3	4	5
1	1.3649e+03	-99.2052	-49.8237	91.8897	110.3587
2	73.3255	67.5331	-26.4304	-56.3956	-12.1745
3	-25.5518	90.1374	-121.2469	98.0200	-47.1906
4	-17.1075	-23.5532	60.3729	-7.3428	-41.6396
5	-22.1086	15.7114	31.0801	1.0475	-1.9229
6	-32.5421	25.7871	0.6717	63.1372	-48.1434
7	18.3436	-16.1884	40.1800	-10.5550	-33.8960
8	29.6503	-21.9424	-7.3973	9.2177	-2.0710
9	13.9812	-4.4391	-11.1338	6.5136	0.9377
10	-7.5909	-2.3892	8.6983	-7.8869	9.2181
11	11.7445	-9.7825	-6.4520	-0.9022	16.1135
12					

Relationship between image size and DCT coefficients

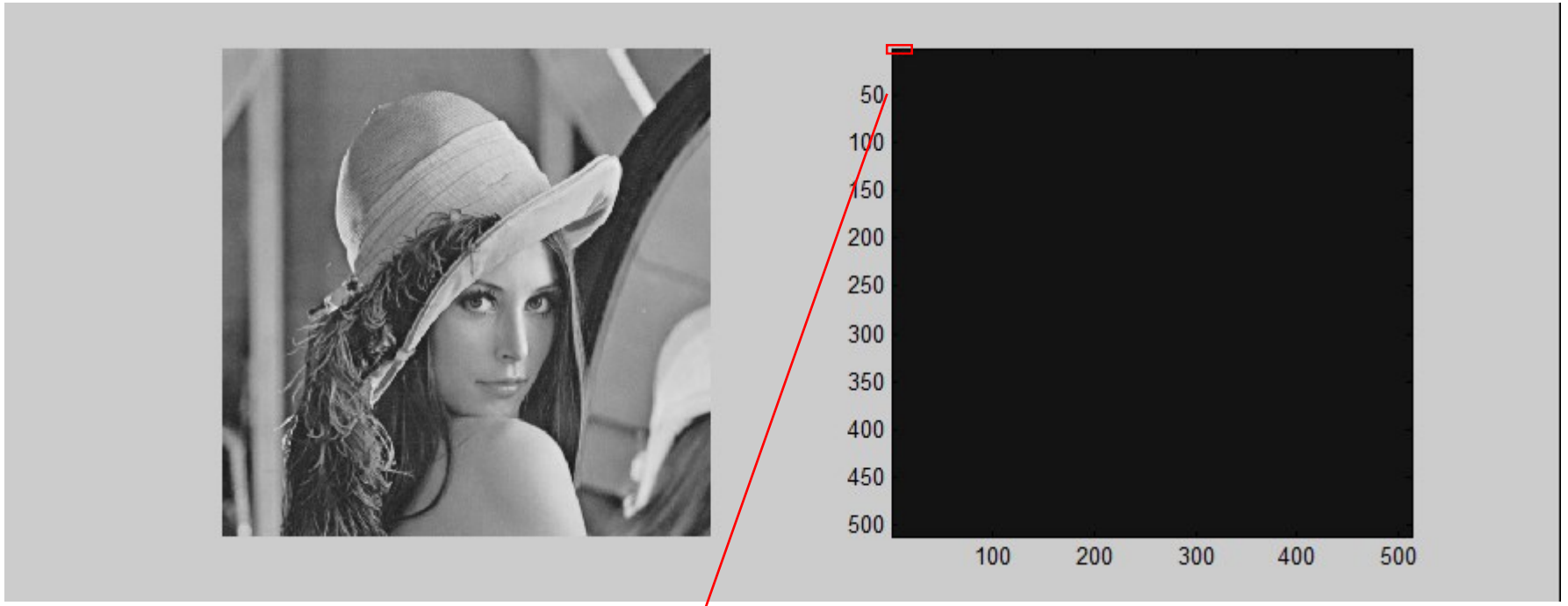
52 x 52



	1	2	3	4	5
1	6.4495e+03	-520.7436	-151.5713	489.1371	450.5086
2	354.8647	383.0446	-286.6618	-161.3687	-98.7322
3	-35.5706	224.3982	-377.7081	447.3406	-304.5452
4	-79.8277	-107.2932	386.7337	-243.9519	-0.6625
5	-132.9977	116.0623	99.6430	102.9810	-10.1339
6	-47.2541	-3.3932	208.0734	105.5907	-247.0023
7	136.9033	-99.6564	157.7367	-121.9128	-91.8851

Relationship between image size and DCT coefficients

512 x 512

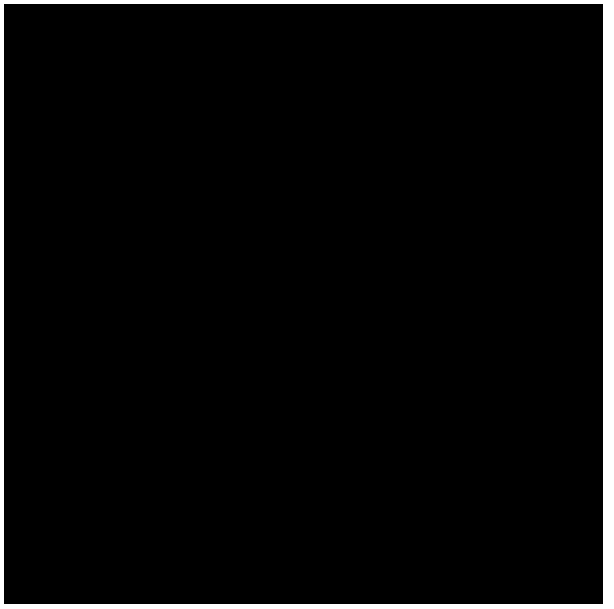


	1	2	3	4	5
1	6.3510e+04	-5.2186e+03	-1.2174e+03	4.9412e+03	4.2037e+03
2	3.5498e+03	3.8991e+03	-3.2492e+03	-1.2073e+03	-1.1665e+03
3	-104.5352	1.5585e+03	-2.9924e+03	4.0129e+03	-2.8571e+03
4	-852.1659	-873.8822	3.8690e+03	-2.8289e+03	618.8543
5	-1.2477e+03	1.0509e+03	1.0615e+03	1.1741e+03	-131.7160
6	-165.8589	-326.9890	2.5260e+03	253.7506	-2.2538e+03
7	1.4370e+03	-1.0205e+03	1.2802e+03	-1.0940e+03	-724.0527
8	1.7526e+03	-1.3310e+03	-833.1556	1.1272e+03	-232.8831
9	-222.0533	168.5836	39.9274	-332.9693	644.6928
10	243.1871	-729.5183	118.0543	-295.0197	978.3837

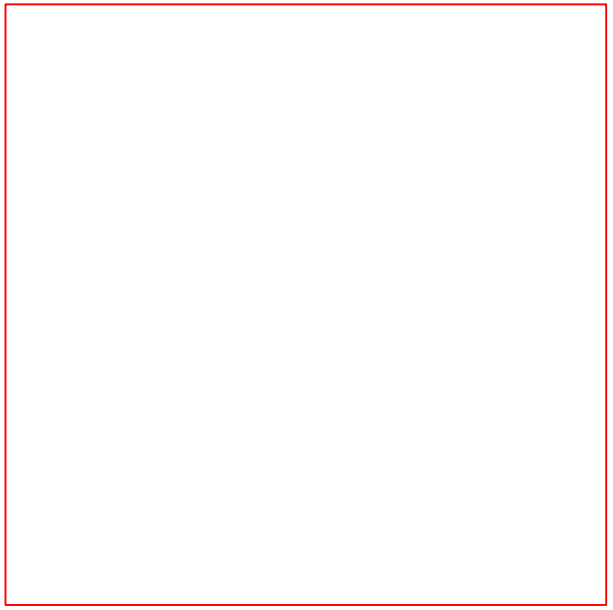
Relationship between image frequency and DCT coefficients

Original Image

Recovered Image



Difference



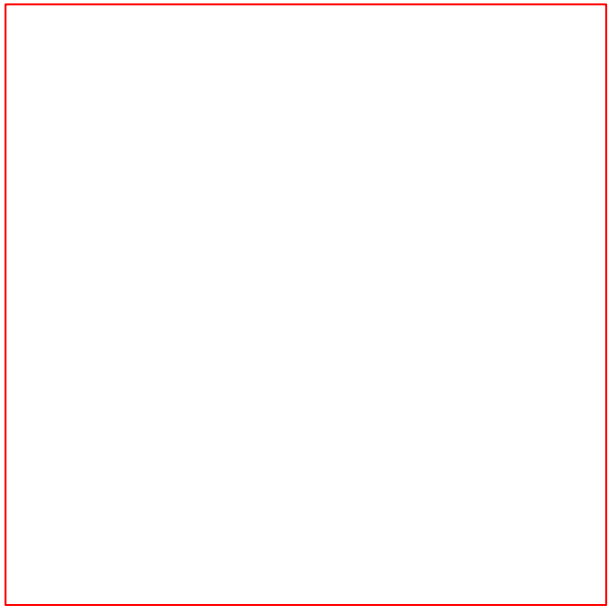
Mask Pattern

Relationship between image frequency and DCT coefficients

Original Image

Recovered Image

Difference



Mask Pattern

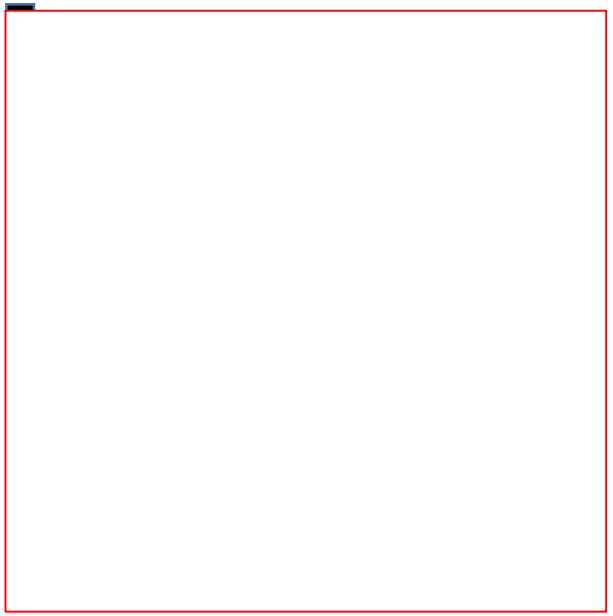
1x1

Relationship between image frequency and DCT coefficients

Original Image

Recovered Image

Difference



Mask Pattern

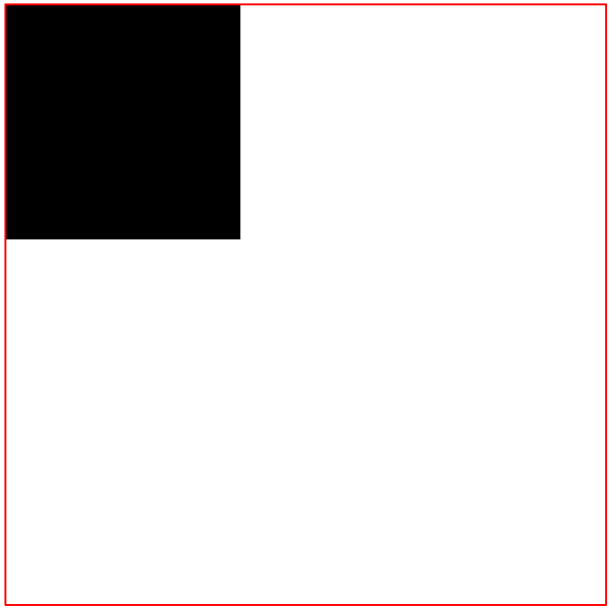
10x10

Relationship between image frequency and DCT coefficients

Original Image

Recovered Image

Difference



Mask Pattern

200x200

Relationship between image frequency and DCT coefficients

Original Image

Recovered Image

Difference

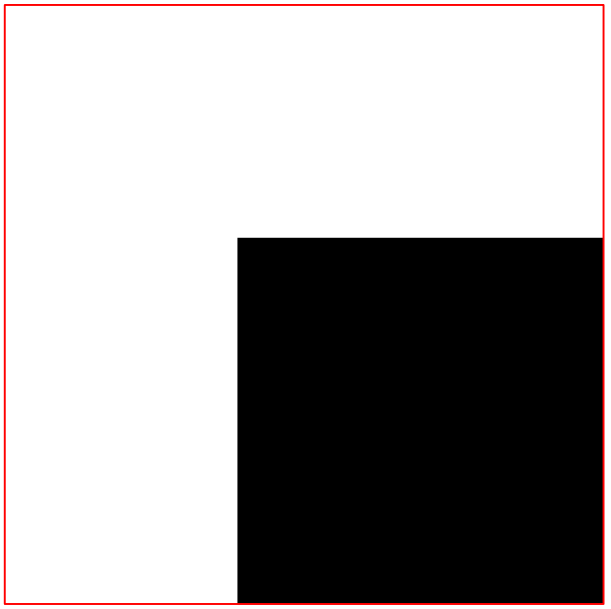
Mask Pattern

Relationship between image frequency and DCT coefficients

Original Image

Recovered Image

Difference



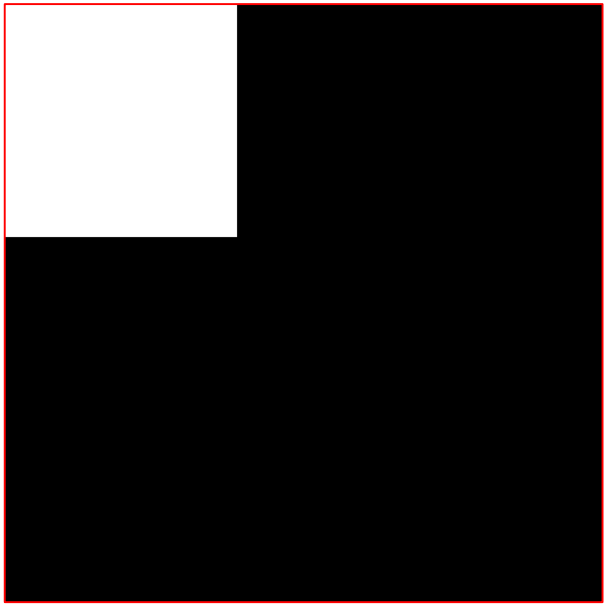
Mask Pattern

Relationship between image frequency and DCT coefficients

Original Image

Recovered Image

Difference

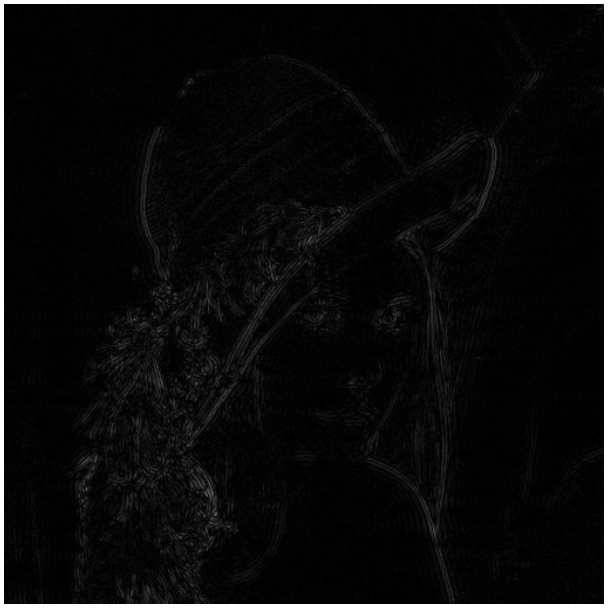


Mask Pattern

Relationship between image frequency and DCT coefficients

Original Image

Recovered Image



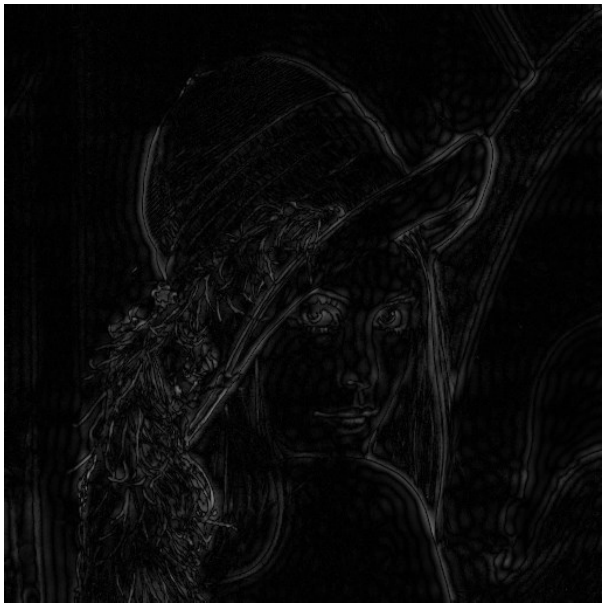
Difference

Mask Pattern

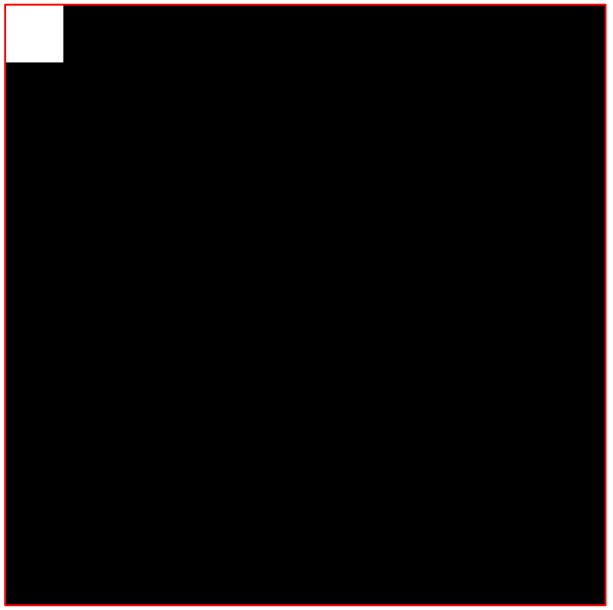
Relationship between image frequency and DCT coefficients

Original Image

Recovered Image



Difference



Mask Pattern

Relationship between image frequency and DCT coefficients

Original Image

Recovered Image

Difference

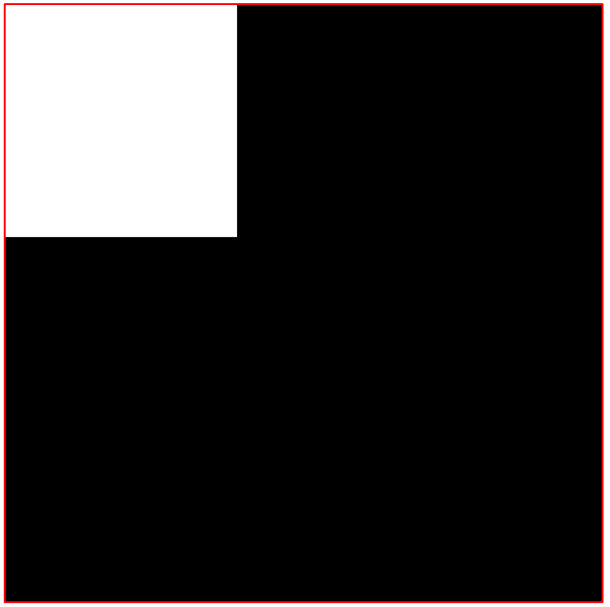
Mask Pattern

Relationship between image frequency and DCT coefficients

Original Image

Recovered Image

Difference



Mask Pattern

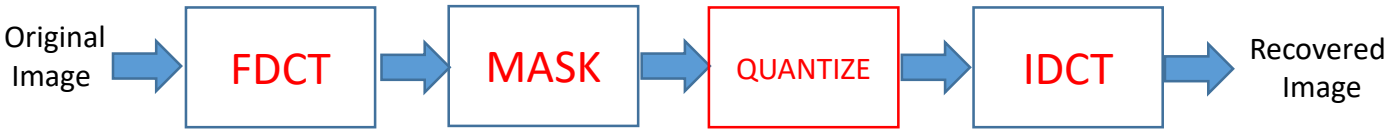
	1	2	3	4
1	6.3510e+04	-5.2185e+03	-1.2172e+03	4.9413e+03
2	3.5497e+03	3.8986e+03	-3.2493e+03	-1.2075e+03
3	-104.6609	1.5583e+03	-2.9925e+03	4.0130e+03
4	-851.8232	-874.1121	3.8686e+03	-2.8288e+03
5	-1.2482e+03	1.0505e+03	1.0611e+03	1.1745e+03
6	-165.7130	-326.5807	2.5264e+03	253.8995
7	1.4371e+03	-1.0204e+03	1.2802e+03	-1.0941e+03
8	1.7531e+03	-1.3305e+03	-833.4106	1.1270e+03
9	-221.9198	168.6203	39.7324	-333.3642
10	243.2396	-729.7188	118.2280	-294.6010
11	-569.8203	529.1188	-532.0261	-46.5159

Relationship between image frequency and DCT coefficients

Original Image

Recovered Image

Difference



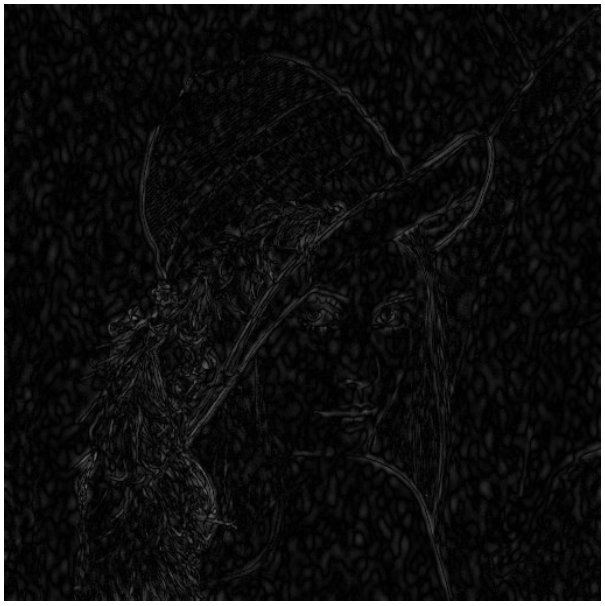
Mask Pattern

24-BIT

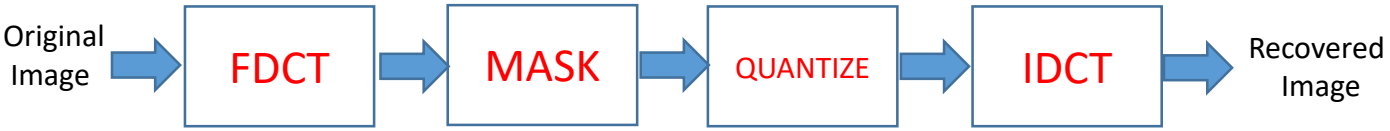
Relationship between image frequency and DCT coefficients

Original Image

Recovered Image



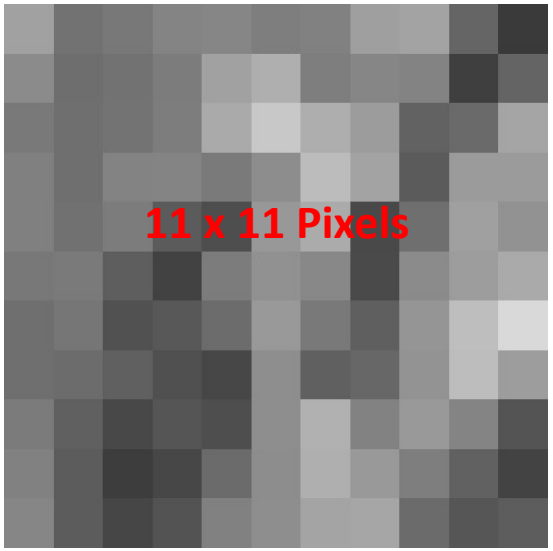
Difference



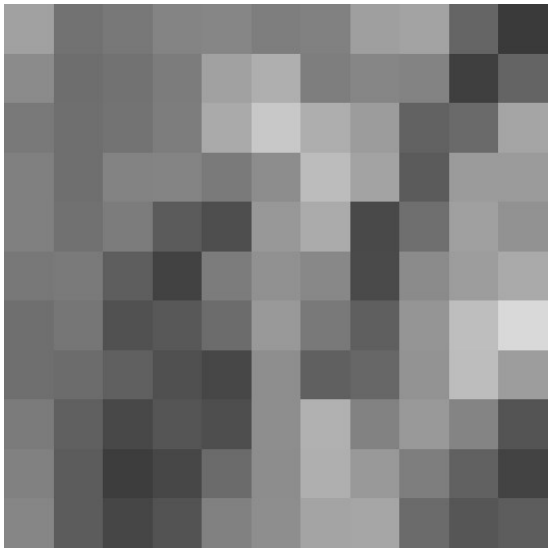
Mask Pattern

8-BIT

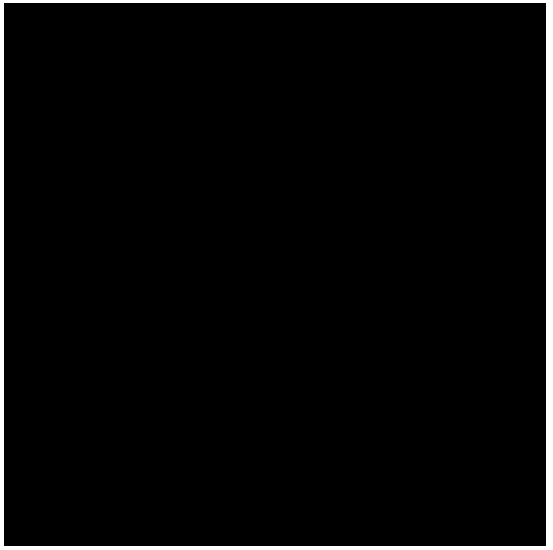
Relationship between image frequency and DCT coefficients



Original Image



Recovered Image

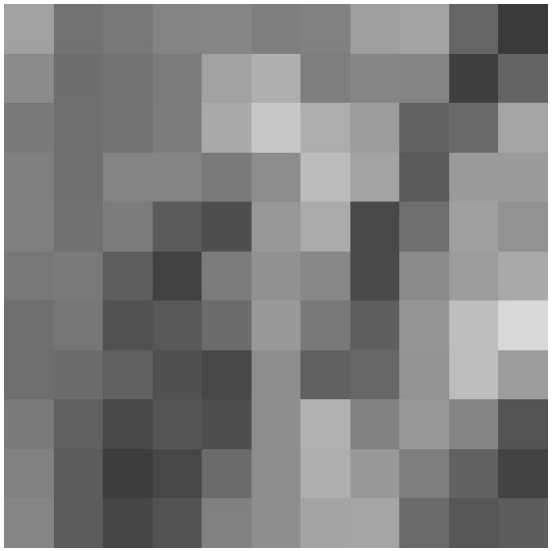


Difference

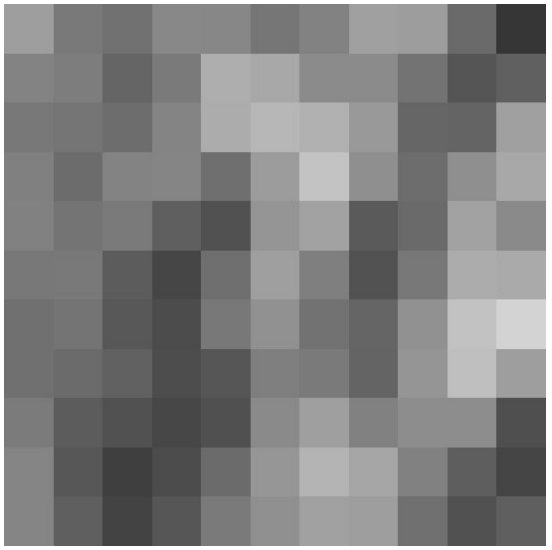
	1	2	3	4	5	6	7	8	9	10	11
1	1.3649e+03	-99.2052	-49.8237	91.8897	110.3587	-3.5995	-32.8883	30.8823	16.9340	-6.5352	2.0509
2	73.3255	67.5331	-26.4304	-56.3956	-12.1745	-3.9397	25.1227	-17.6125	20.8615	11.8741	3.7422
3	-25.5518	90.1374	-121.2469	98.0200	-47.1906	43.4739	75.3813	-38.7571	9.6894	13.6114	-3.3104
4	-17.1075	-23.5532	60.3729	-7.3428	-41.6396	62.7329	-40.2204	-10.7158	27.2102	-24.4389	-12.0919
5	-22.1086	15.7114	31.0801	1.0475	-1.9229	29.4013	-11.0035	-13.8581	1.8397	-8.1757	25.4932
6	-32.5421	25.7871	0.6717	63.1372	-48.1434	0.1742	-14.5857	61.4364	-11.9249	-9.5090	15.3389
7	18.3436	-16.1884	40.1800	-10.5550	-33.8960	-11.5371	13.7111	26.9295	-13.2509	23.5859	-25.6042
8	29.6503	-21.9424	-7.3973	9.2177	-2.0710	-2.3186	26.0117	-6.1949	-26.8234	7.2502	-0.7330
9	13.9812	-4.4391	-11.1338	6.5136	0.9377	1.4823	3.8832	-5.6331	-8.9524	1.9252	8.2897
10	-7.5909	-2.3892	8.6983	-7.8869	9.2181	-0.3284	-19.5435	-1.7107	7.5322	7.8304	-5.3475
11	11.7445	-9.7825	-6.4520	-0.9022	16.1135	-11.5351	-4.2073	14.3701	4.8269	-16.6809	-0.4980

Discrete Cosine Coefficient

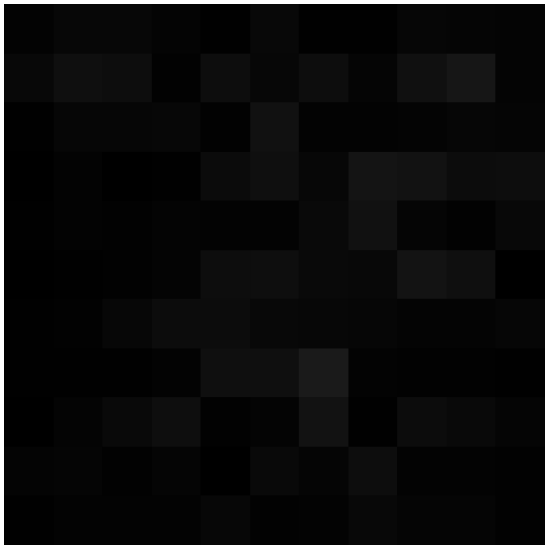
Relationship between image frequency and DCT coefficients



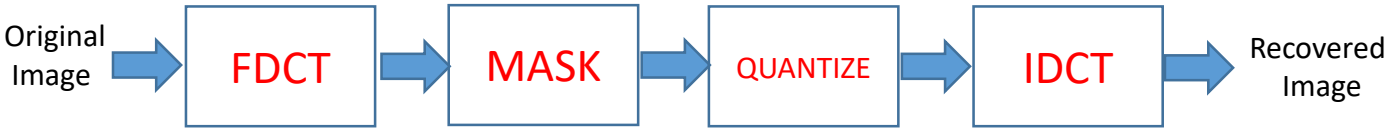
Original Image



Recovered Image



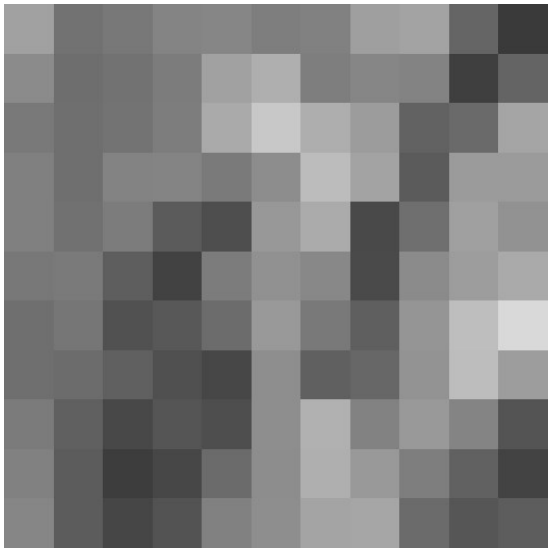
Difference



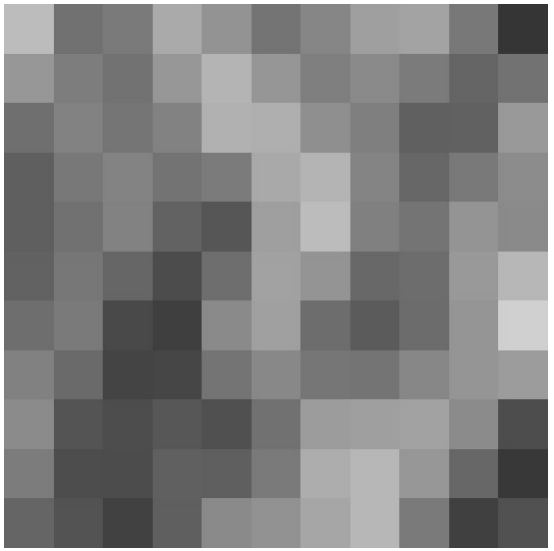
Mask Pattern

8-BIT

Relationship between image frequency and DCT coefficients

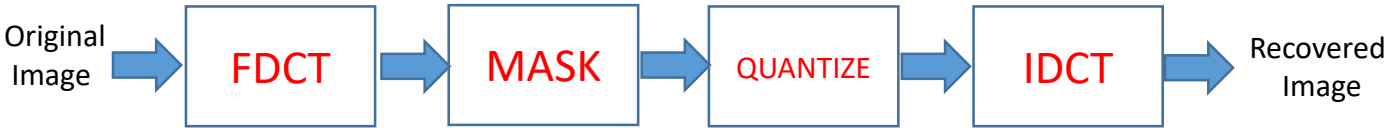


Original Image



Recovered Image

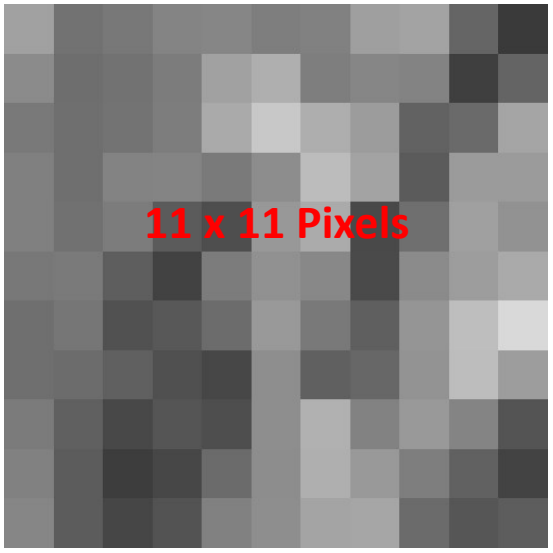
Difference



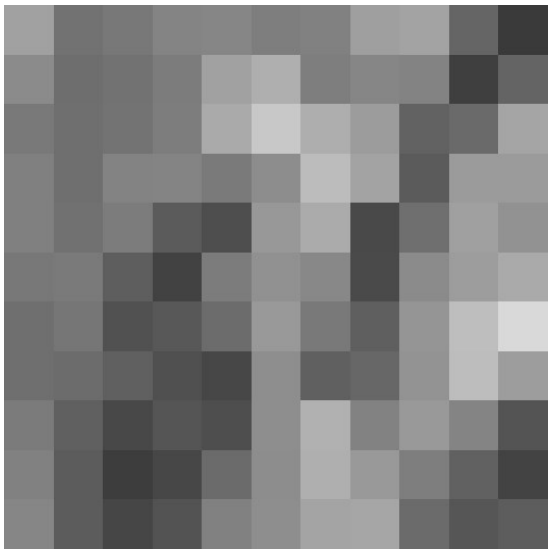
Mask Pattern

4-BIT

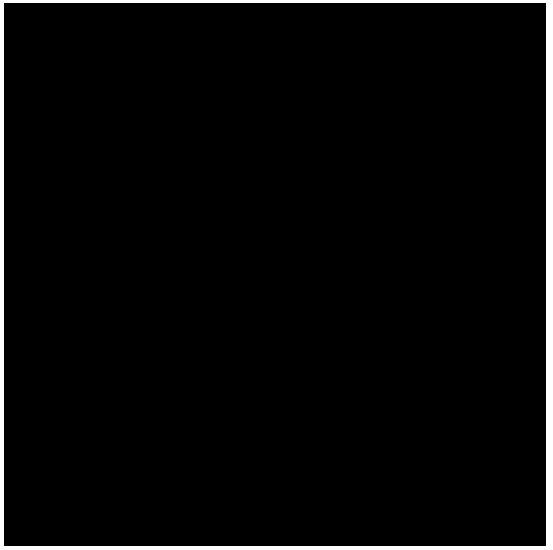
Relationship between image frequency and DCT coefficients



Original Image



Recovered Image



Difference

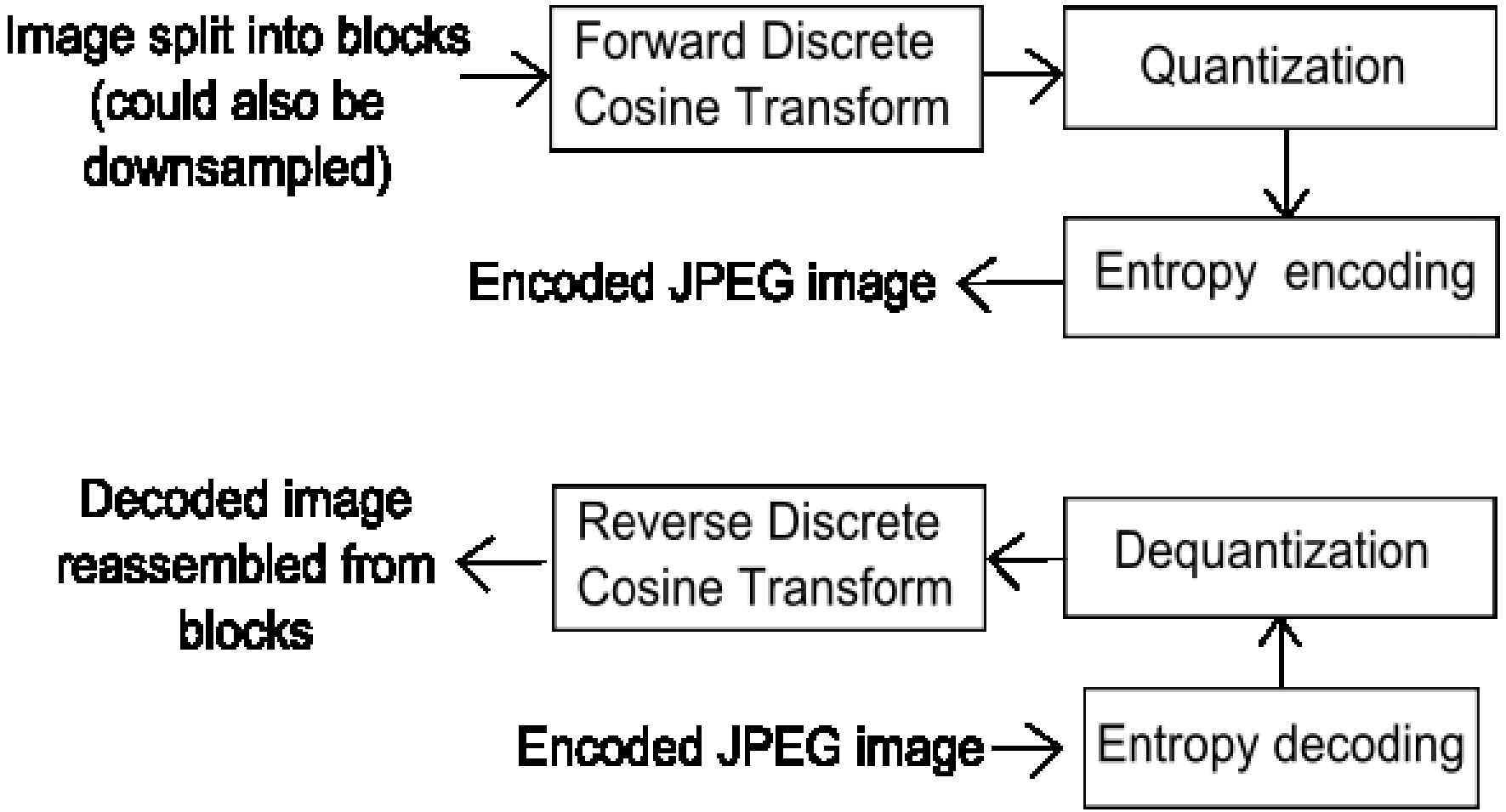
	1	2	3	4	5	6	7	8	9	10	11
1	1.3649e+03	-99.2052	-49.8237	91.8897	110.3587	-3.5995	-32.8883	30.8823	16.9340	-6.5352	2.0509
2	-73.[illegible]	87.[illegible]	-26.[illegible]	-56.3956	-12.1745	-3.9397	25.1227	-17.6125	20.8615	11.8741	3.7422
3	-25.5518	90.1374	-121.2469	98.0200	-47.1906	43.4739	75.3813	-38.7571	9.6894	13.6114	-3.3104
4	-17.1075	-23.5532	60.3729	-7.3428	-41.6396	62.7329	-40.2204	-10.7158	27.2102	-24.4389	-12.0919
5	-22.1086	15.7114	31.0801	1.0475	-1.9229	29.4013	-11.0035	-13.8581	1.8397	-8.1757	25.4932
6	-32.5421	25.7871	0.6717	63.1372	-48.1434	0.1742	-14.5857	61.4364	-11.9249	-9.5090	15.3389
7	18.3436	-16.1884	40.1800	-10.5550	-33.8960	-11.5371	13.7111	26.9295	-13.2509	23.5859	-25.6042
8	29.6503	-21.9424	-7.3973	9.2177	-2.0710	-2.3186	26.0117	-6.1949	-26.8234	7.2502	-0.7330
9	13.9812	-4.4391	-11.1338	6.5136	0.9377	1.4823	3.8832	-5.6331	-8.9524	1.9252	8.2897
10	-7.5909	-2.3892	8.6983	-7.8869	9.2181	-0.3284	-19.5435	-1.7107	7.5322	7.8304	-5.2475
11	11.7445	-9.7825	-6.4520	-0.9022	16.1135	-11.5351	-4.2073	14.3701	4.8269	-16.6809	-0.4980

High impact

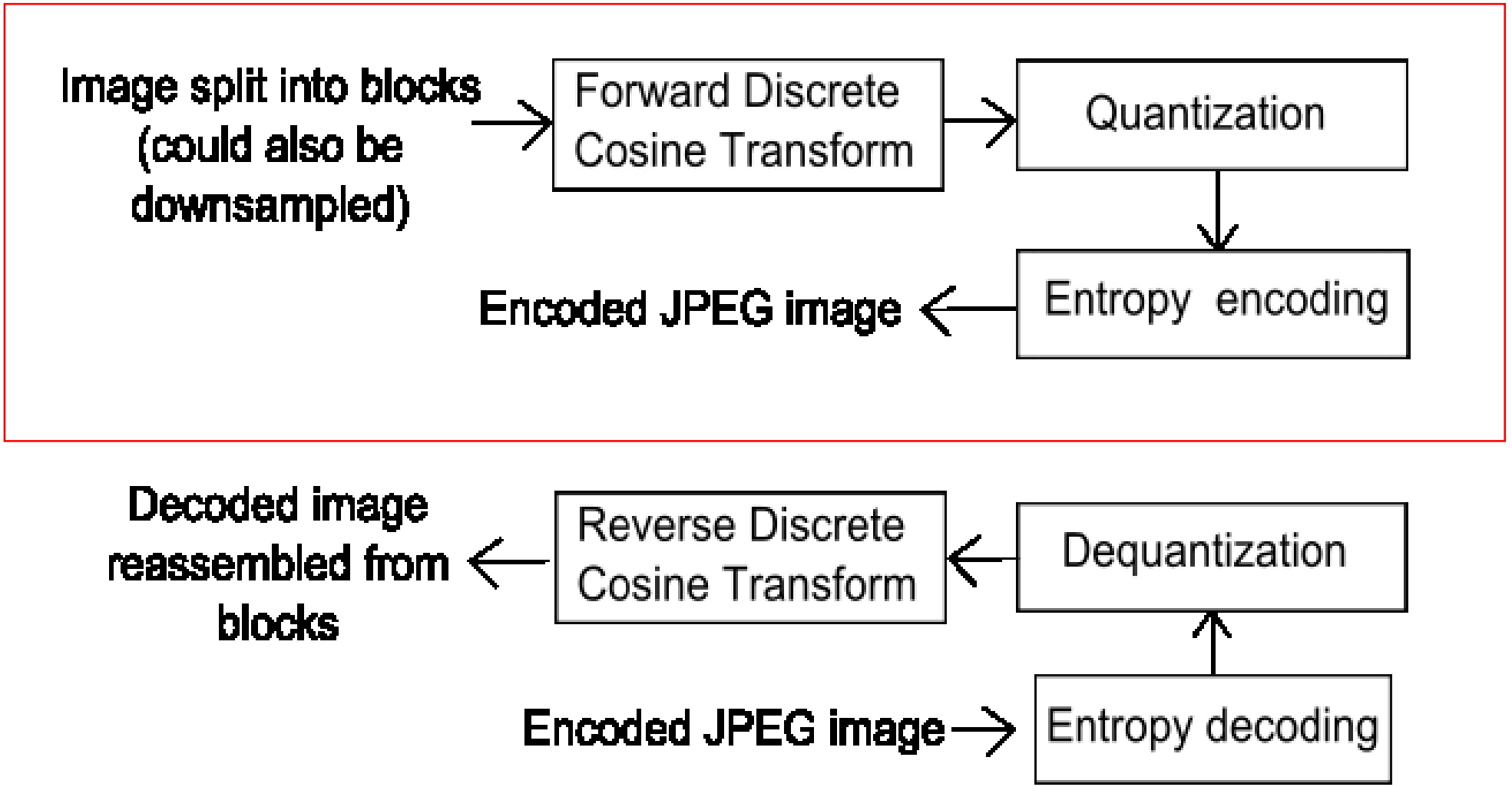
Low impact

Discrete Cosine Coefficient

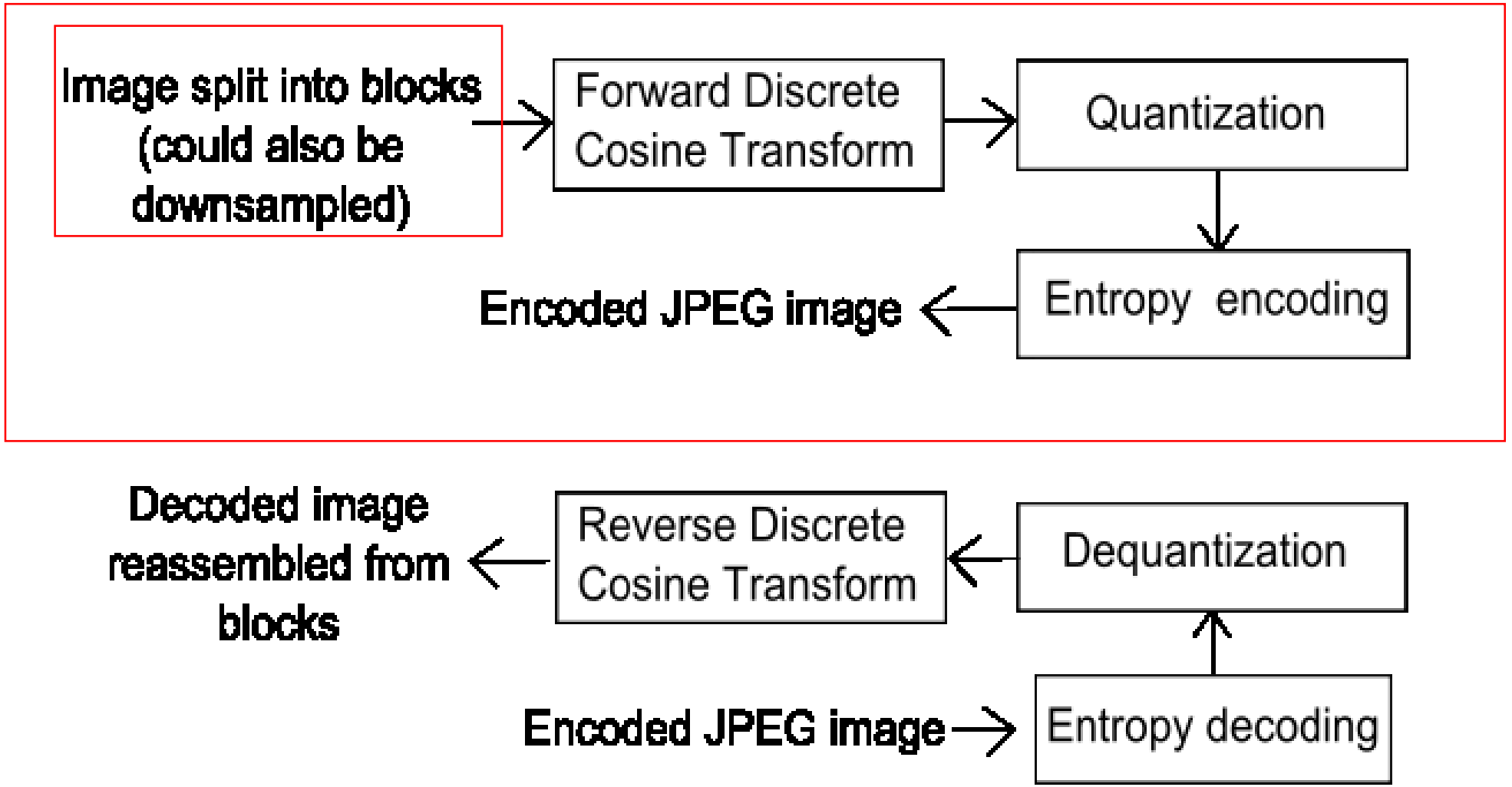
JPEG Image Compression Algorithm



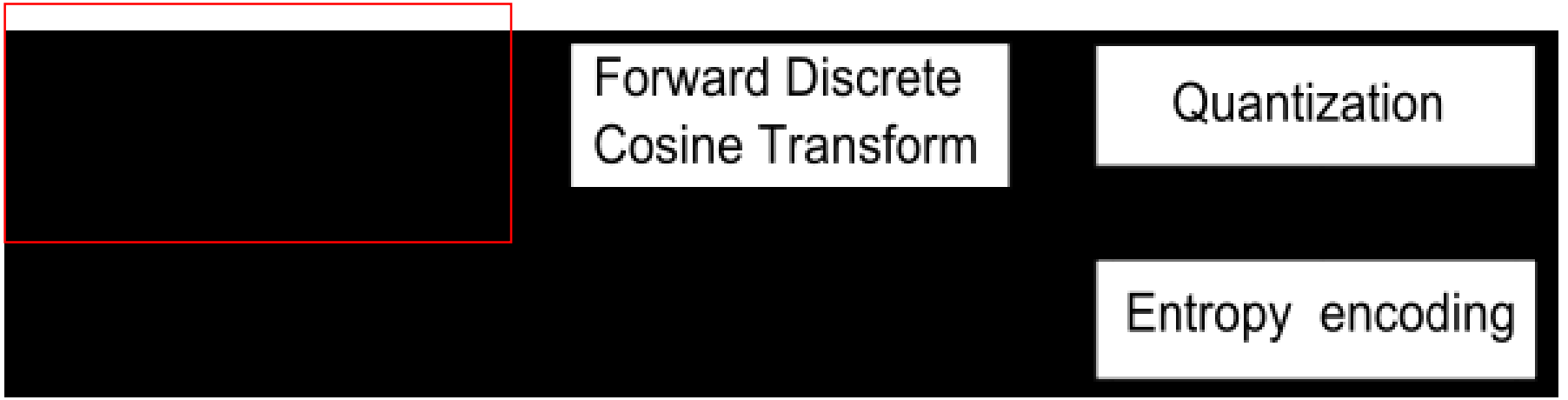
JPEG Image Compression Algorithm



JPEG Image Compression Algorithm



JPEG Image Compression Algorithm



JPEG Image Compression Algorithm

98	91	84	78	74	83	78	84
97	94	81	81	72	75	78	75
96	89	79	74	75	74	80	85
95	86	79	78	74	71	77	81
101	90	82	83	74	70	78	75
99	86	85	80	72	69	76	86
95	83	87	71	73	70	81	78
99	81	85	81	69	68	75	78

FDCT

649.5000	44.7806	40.2668	-0.8119	8.2500	9.3513	1.1804	1.7943
7.5876	-2.5360	-0.1526	6.3021	-4.5284	-7.6330	-2.8671	-8.1616
1.0128	-0.2040	-0.6731	0.2222	-4.3952	1.2259	5.8247	-0.5401
5.8321	2.0361	-0.2271	-0.7432	-0.9167	-0.4781	2.2782	-1.0104
0.5000	2.4496	-3.0026	1.2275	2.7500	2.6562	1.2437	-3.3634
-1.0598	-8.8306	2.5785	0.4413	-2.2920	-4.1585	3.8596	5.2484
3.0983	-4.1411	4.0747	-2.8049	5.0677	-2.1182	3.9231	-1.6159
1.6330	-0.2422	0.5145	3.4207	-3.0455	-0.3569	-0.9772	2.9376

Forward Discrete Cosine Transform

Quantization

Entropy encoding

JPEG Image Compression Algorithm

649.5000	44.7806	40.2668	-0.8119	8.2500	9.3513	1.1804	1.7943
7.5876	-2.5360	-0.1526	6.3021	-4.5284	-7.6330	-2.8671	-8.1616
1.0128	-0.2040	-0.6731	0.2222	-4.3952	1.2259	5.8247	-0.5401
5.8321	2.0361	-0.2271	-0.7432	-0.9167	-0.4781	2.2782	-1.0104
0.5000	2.4496	-3.0026	1.2275	2.7500	2.6562	1.2437	-3.3634
-1.0598	-8.8306	2.5785	0.4413	-2.2920	-4.1585	3.8596	5.2484
3.0983	-4.1411	4.0747	-2.8049	5.0677	-2.1182	3.9231	-1.6159
1.6330	-0.2422	0.5145	3.4207	-3.0455	-0.3569	-0.9772	2.9376

DCT

$\odot$

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	36	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Quantization matrix

$^{-1}$

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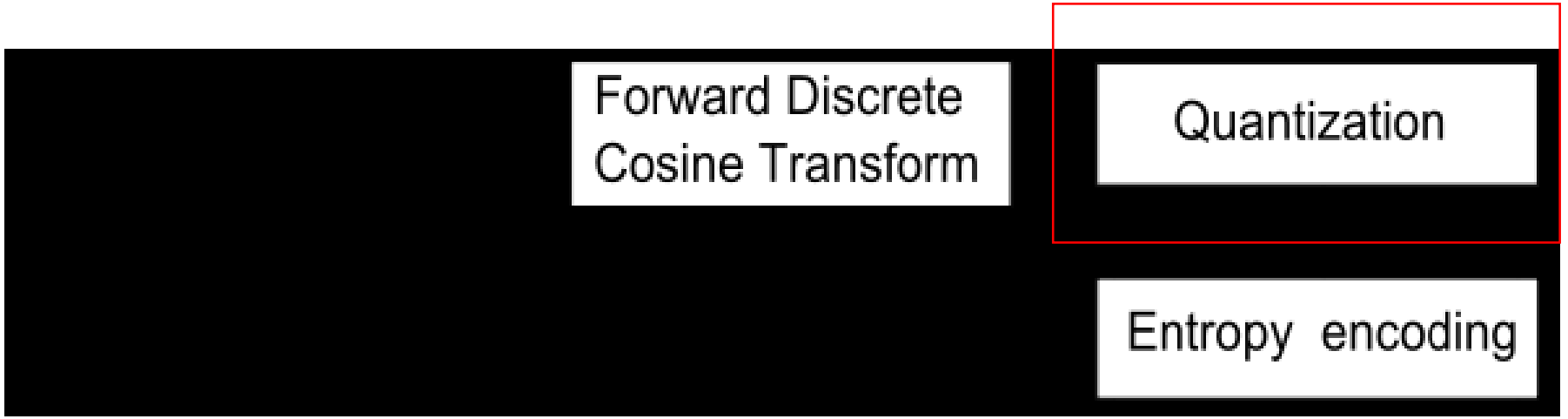
41	4	4	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Quantized DCT



JPEG Image Compression Algorithm : Quantization Matrix

Quantization Table for: Photoshop - (Save As 12)
The following matrix can be used to evaluate the JPEG compression quality of Photoshop's .

Chroma Subsampling: 1x1

Quantization Table: Luminance							
1	1	1	1	1	1	1	2
1	1	1	1	1	1	1	2
1	1	1	1	1	1	2	2
1	1	1	1	1	2	2	3
1	1	1	1	2	2	3	3
1	1	1	2	2	3	3	3
1	1	2	2	3	3	3	3
2	2	2	3	3	3	3	3

Quantization Table: Chrominance							
1	1	1	2	3	3	3	3
1	1	1	2	3	3	3	3
1	1	2	3	3	3	3	3
2	2	3	3	3	3	3	3
3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3

Quantization Table for: Photoshop - (Save As 09)
The following matrix can be used to evaluate the JPEG compression quality of Photoshop's .

Chroma Subsampling: 1x1

Quantization Table: Luminance							
4	3	4	7	9	11	14	17
3	3	4	7	9	12	12	12
4	4	5	9	12	12	12	12
7	7	9	12	12	12	12	12
9	9	12	12	12	12	12	12
11	12	12	12	12	12	12	12
14	12	12	12	12	12	12	12
17	12	12	12	12	12	12	12

Quantization Table: Chrominance							
4	6	12	22	20	20	17	17
6	8	12	14	14	12	12	12
12	12	14	14	12	12	12	12
22	14	14	12	12	12	12	12
20	14	12	12	12	12	12	12
20	12	12	12	12	12	12	12
17	12	12	12	12	12	12	12
17	12	12	12	12	12	12	12

Quantization Table for: Photoshop - (Save As 04)
The following matrix can be used to evaluate the JPEG compression quality of Photoshop's .

Chroma Subsampling: 2x2

Quantization Table: Luminance							
16	11	17	27	34	39	34	17
11	12	16	26	28	23	12	12
17	16	21	28	23	12	12	12
27	26	28	23	12	12	12	12
34	28	23	12	12	12	12	12
39	23	12	12	12	12	12	12
34	12	12	12	12	12	12	12
17	12	12	12	12	12	12	12

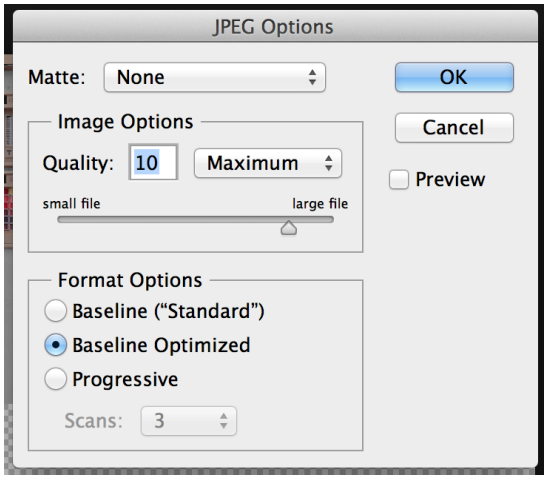
Quantization Table: Chrominance							
17	17	22	34	20	20	17	17
17	19	22	14	14	12	12	12
22	22	14	14	12	12	12	12
34	14	14	12	12	12	12	12
20	14	12	12	12	12	12	12
20	12	12	12	12	12	12	12
17	12	12	12	12	12	12	12
17	12	12	12	12	12	12	12

Quantization Table for: Photoshop - (Save As 00)
The following matrix can be used to evaluate the JPEG compression quality of Photoshop's .

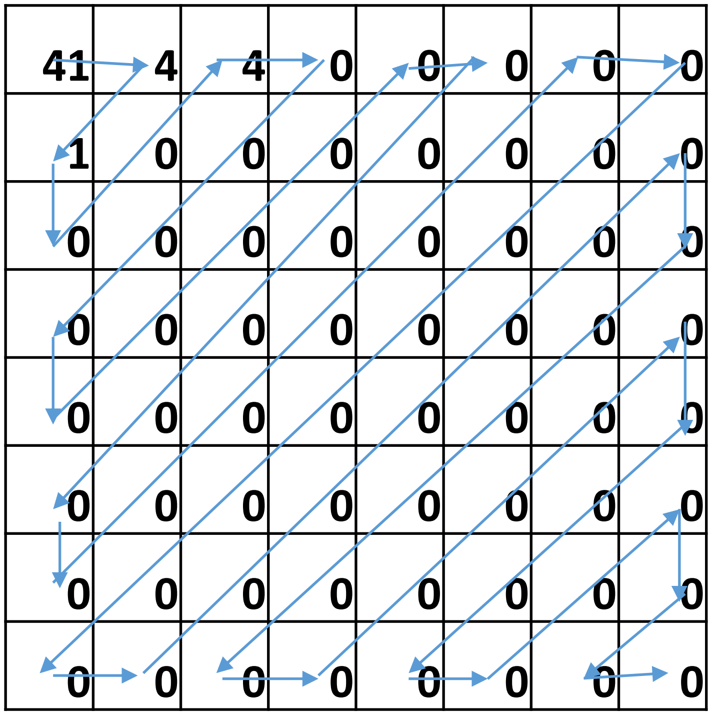
Chroma Subsampling: 2x2

Quantization Table: Luminance							
32	33	51	81	66	39	34	17
33	36	48	47	28	23	12	12
51	48	47	28	23	12	12	12
81	47	28	23	12	12	12	12
66	28	23	12	12	12	12	12
39	23	12	12	12	12	12	12
34	12	12	12	12	12	12	12
17	12	12	12	12	12	12	12

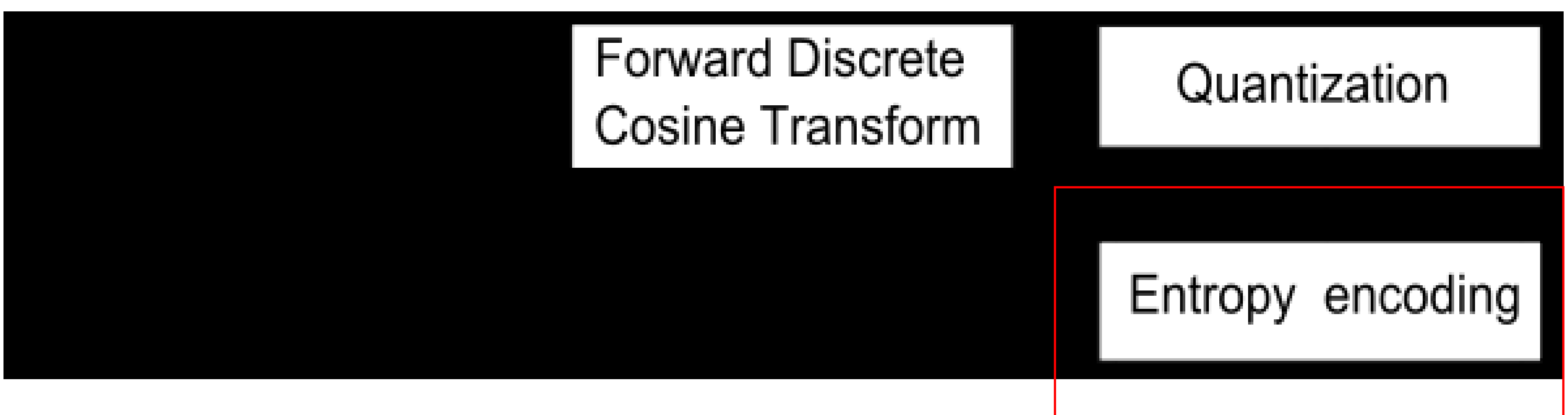
Quantization Table: Chrominance							
34	51	52	34	20	20	17	17
51	38	24	14	14	12	12	12
52	24	14	14	12	12	12	12
34	14	14	12	12	12	12	12
20	14	12	12	12	12	12	12
20	12	12	12	12	12	12	12
17	12	12	12	12	12	12	12
17	12	12	12	12	12	12	12



JPEG Image Compression Algorithm



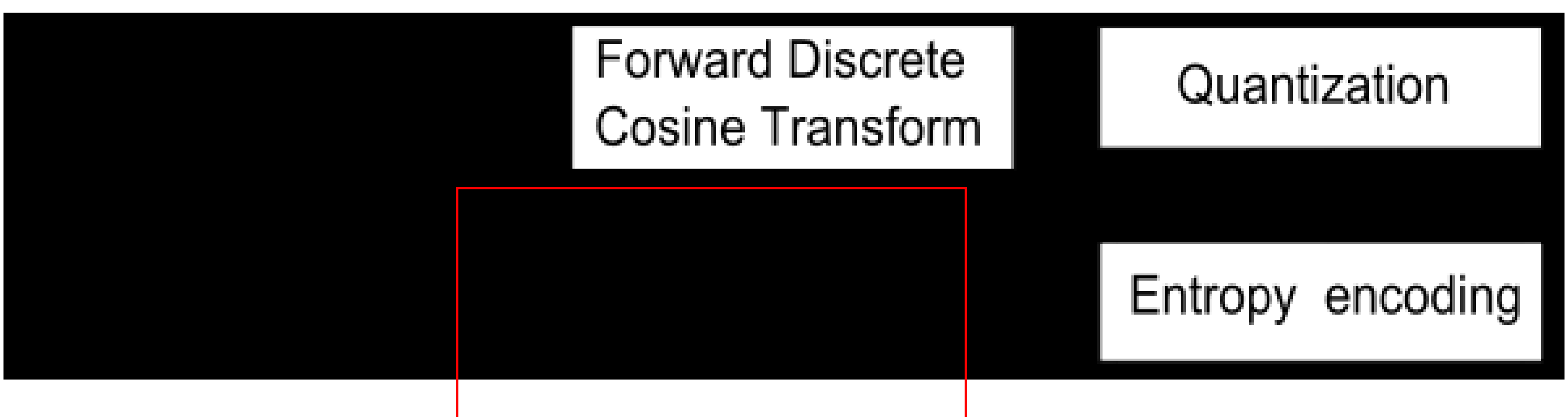
41,4,1,0,0,4,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,0,0



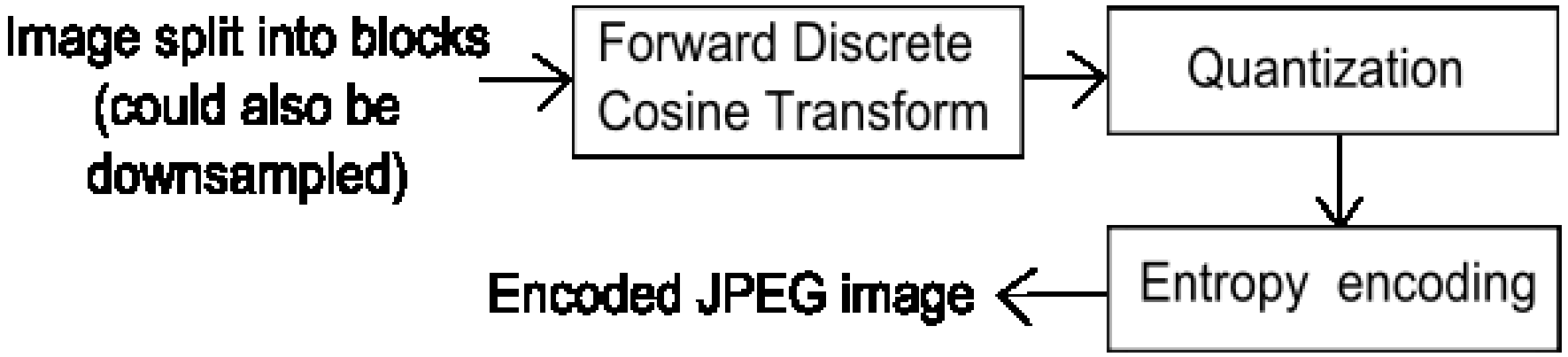
JPEG Image Compression Algorithm

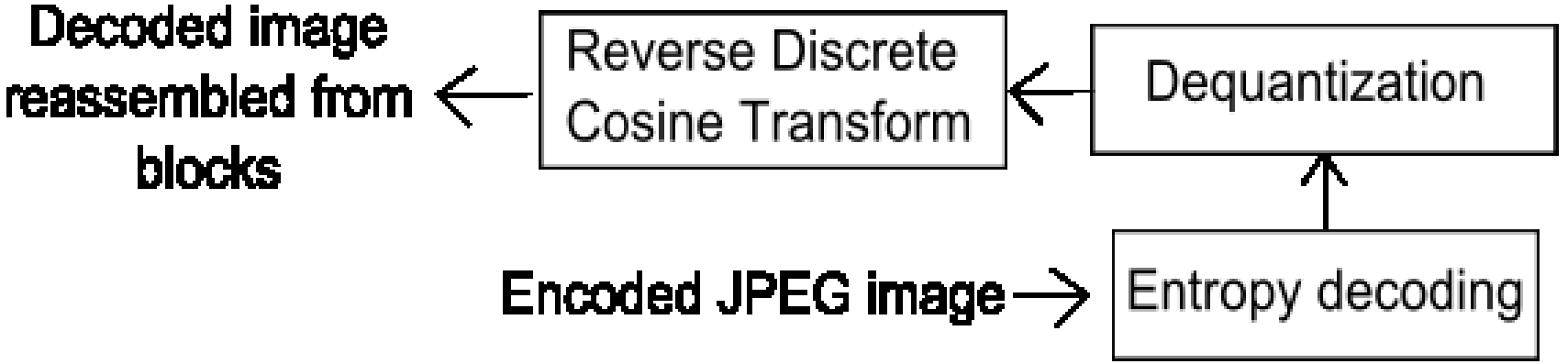
41,4,1,0,0,4,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0

141114112014580



JPEG Image Compression Algorithm





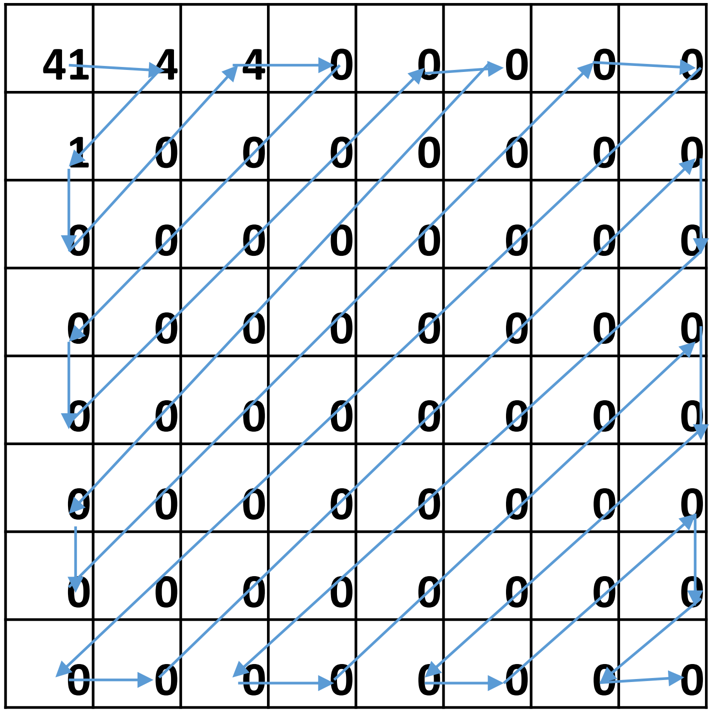
JPEG Image Compression Algorithm

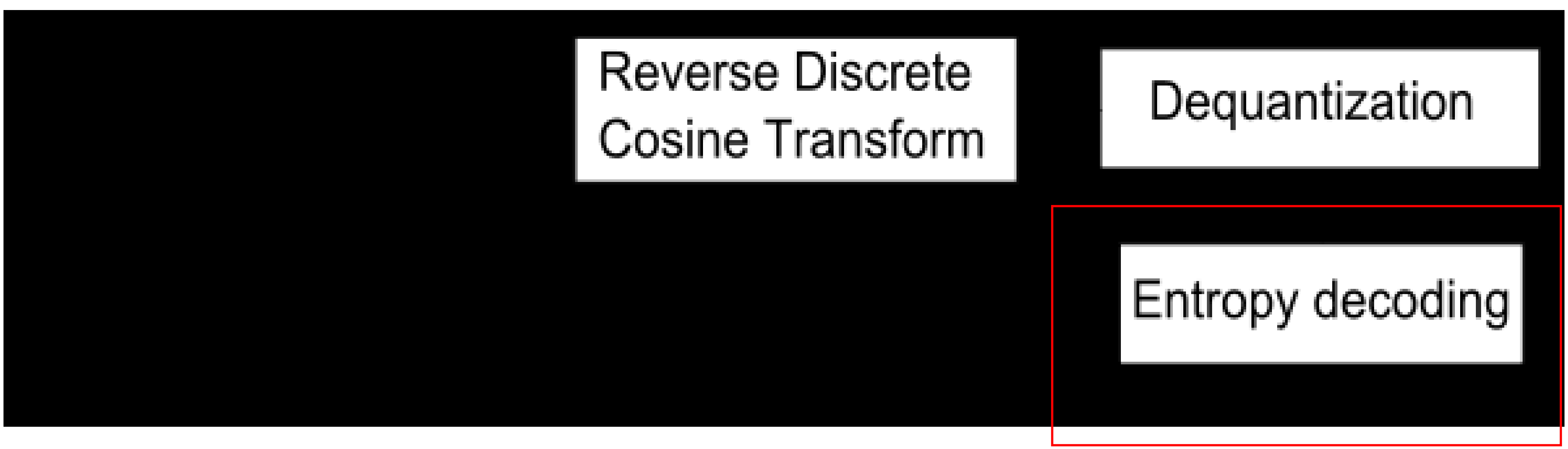
141114112014580

**41,4,1,0,0,4,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,0,0**

JPEG Image Compression Algorithm

41,4,1,0,0,4,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0

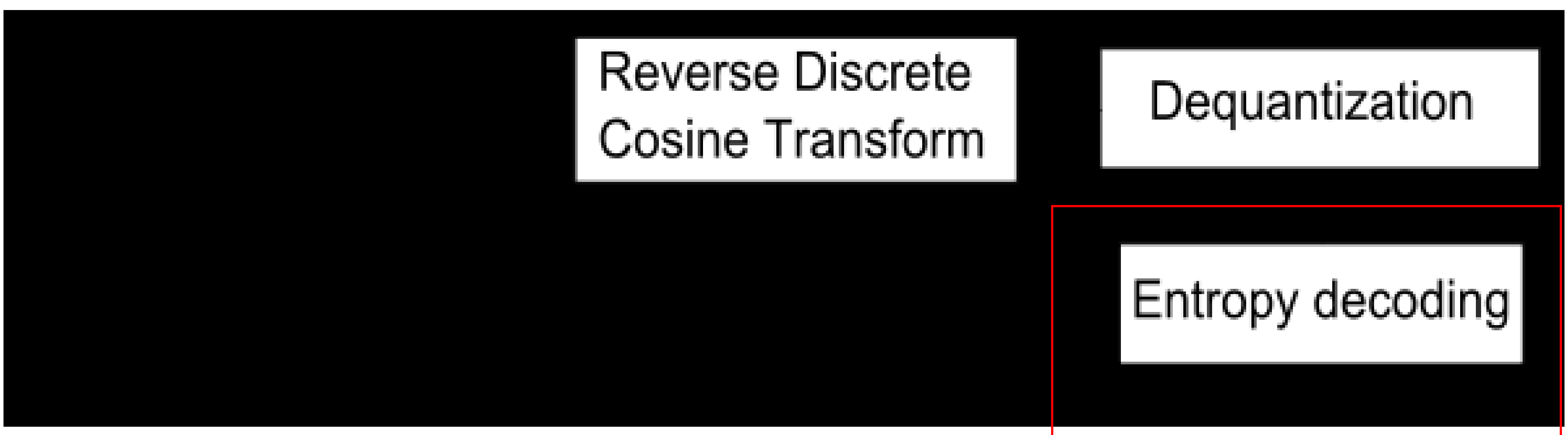




JPEG Image Compression Algorithm

41,4,1,0,0,4,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,0,0

41	4	4	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



JPEG Image Compression Algorithm

41	4	4	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

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16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	36	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

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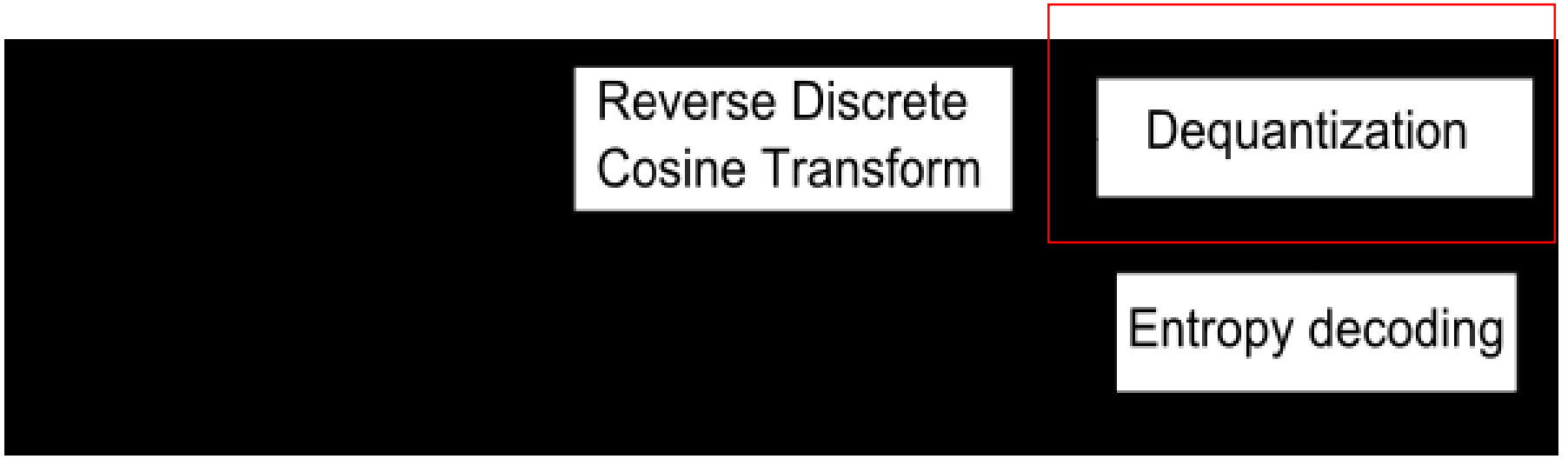
656	44	40	0	0	0	0	0
12	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



JPEG Image Compression Algorithm

656	44	40	0	0	0	0	0
12	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

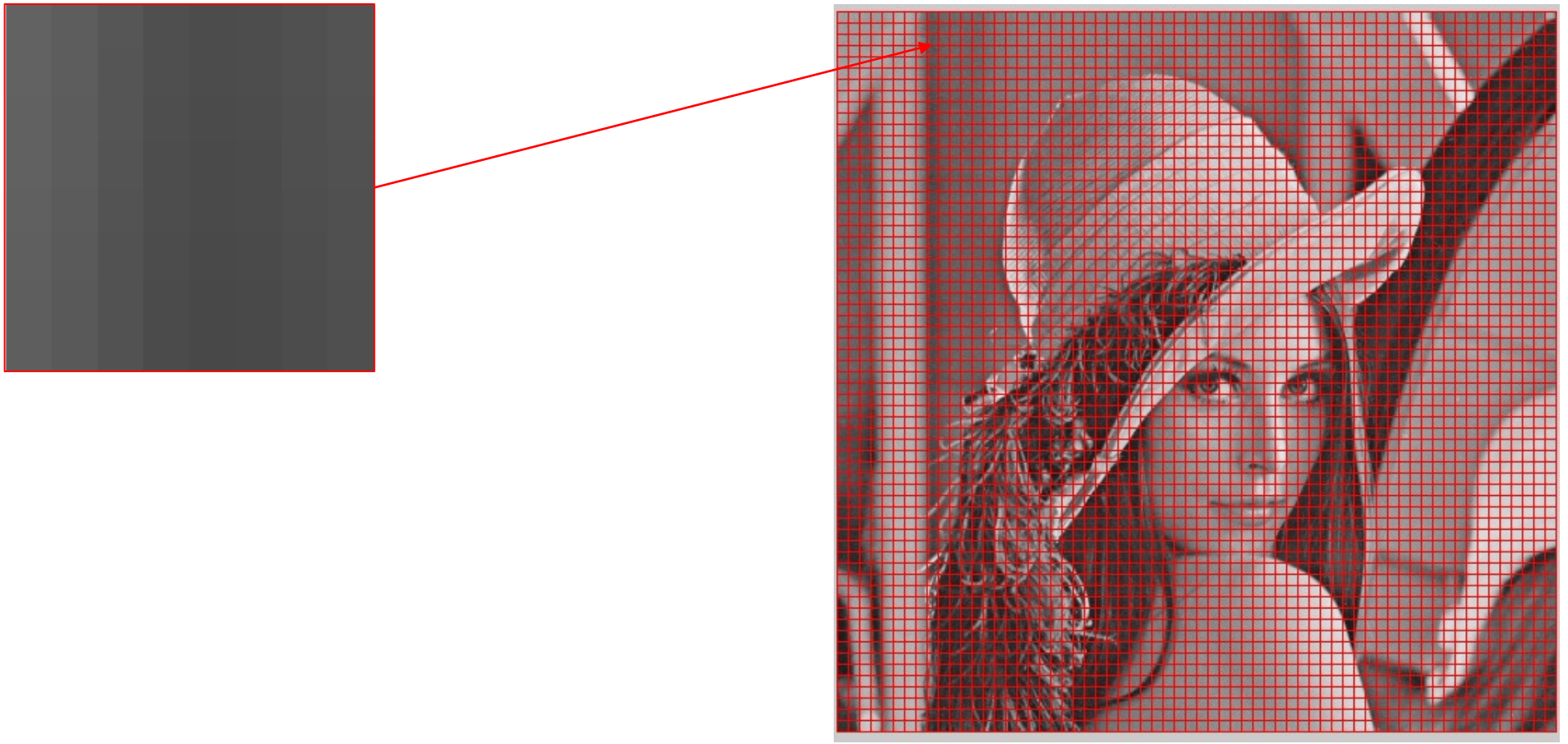
IDCT

98	93	86	79	76	77	80	83
98	93	85	79	76	77	80	83
97	92	85	78	75	76	79	82
97	92	84	77	74	75	79	81
96	91	83	77	74	75	78	80
95	90	82	76	73	74	77	80
94	89	82	75	72	73	76	79
94	89	82	75	72	73	76	79

Original Image

98 91 84 78 74 83 78 84
97 94 81 81 72 75 78 75
96 89 79 74 75 74 80 85
95 86 79 78 74 71 77 81
101 90 82 83 74 70 78 75
99 86 85 80 72 69 76 86
95 83 87 71 73 70 81 78
99 81 85 81 69 68 75 78

JPEG Image Compression Algorithm



Reverse Discrete Cosine Transform

Dequantization

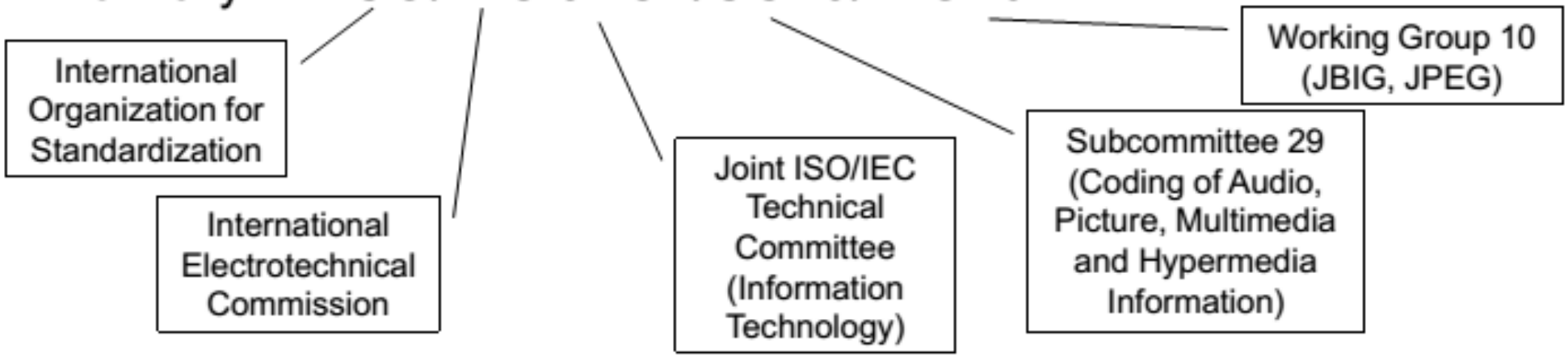
Entropy decoding

JPEG Image Compression Algorithm

JPEG: image partition into 8x8 block

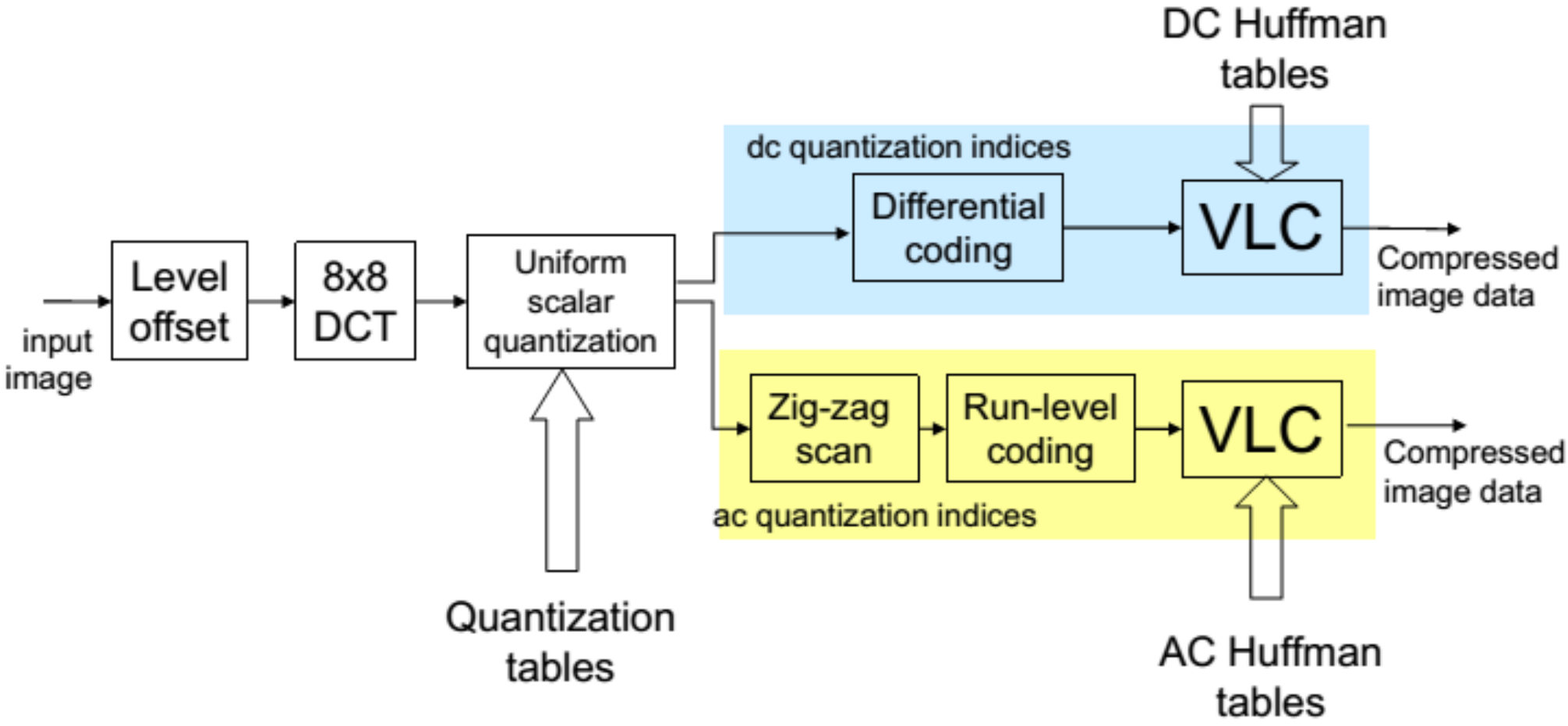
JPEG standard

- JPEG: "Joint Photographic Experts Group"
- Formally: ISO/IEC JTC1/SC29/WG10



- Joint effort with CCITT (International Telephone and Telegraph Consultative Committee, now ITU-T) Study Group VIII
- Work commenced in 1986
- International standard ISO/IEC 10918-1 and CCITT Rec. T.81 in 1992
- Widely used for image exchange, WWW, and digital photography
- Motion-JPEG is de facto standard for digital video editing

Baseline JPEG coder



Common JPEG markers

Short name	Bytes	Payload	Name	Comments
SOI	0xFF, 0xD8	*none*	Start Of Image	
SOF0	0xFF, 0xC0	*variable size*	Start Of Frame (Baseline DCT)	Indicates that this is a baseline DCT-based JPEG, and specifies the width, height. , number of components, and component subsampling (e.g., 4:2:0)
SOF2	0xFF, 0xC2	*variable size*	Start Of Frame (Progressive DCT)	Indicates that this is a progressive DCT-based JPEG, and specifies the width, height, number of components, and component subsampling (e.g., 4:2:0).
DHT	0xFF, 0xC4	*variable size*	Define Huffman Table(s)	Specifies one or more Huffman tables.
DQT	0xFF, 0xDB	*variable size*	Define Quantization Table(s)	Specifies one or more quantization tables.
DRI	0xFF, 0xDD	4 bytes	Define Restart Interval	Specifies the interval between RST*n* markers, in macroblocks. This marker is followed by two bytes indicating the fixed size so it can be treated like any other variable size segment.
SOS	0xFF, 0xDA	*variable size*	Start Of Scan	Begins a top-to-bottom scan of the image. In baseline DCT JPEG images, there is generally a single scan. Progressive DCT JPEG images usually contain multiple scans. This marker specifies which slice of data it will contain, and is immediately followed by entropy-coded data.
RST*n*	0xFF, 0xD*n*(*n*=0..7)	*none*	Restart	Inserted every *r* macroblocks, where *r* is the restart interval set by a DRI marker. Not used if there was no DRI marker. The low 3 bits of the marker code cycle in value from 0 to 7.
APP*n*	0xFF, 0xE*n*	*variable size*	Application-specific	For example, an Exif JPEG file uses an APP1 marker to store metadata, laid out in a structure based closely on TIFF.
COM	0xFF, 0xFE	*variable size*	Comment	Contains a text comment.
EOI	0xFF, 0xD9	*none*	End Of Image	

Matlab image processing function

Forward Discrete Cosine Transform:

Idct=dct2**(im)**;

Inverse Discrete Cosine Transform:

im=idct2(**Idct**);

Image Compression Demo

http://insy.ewi.tudelft.nl/content/image-and-video-compression-learning-tool-vcdemo