DTI 516 Multimedia Processing

Chapter: 10

Frequency Analysis

Dr. Paween Khoenkaw
Digital Technology Innovation : Maejo University

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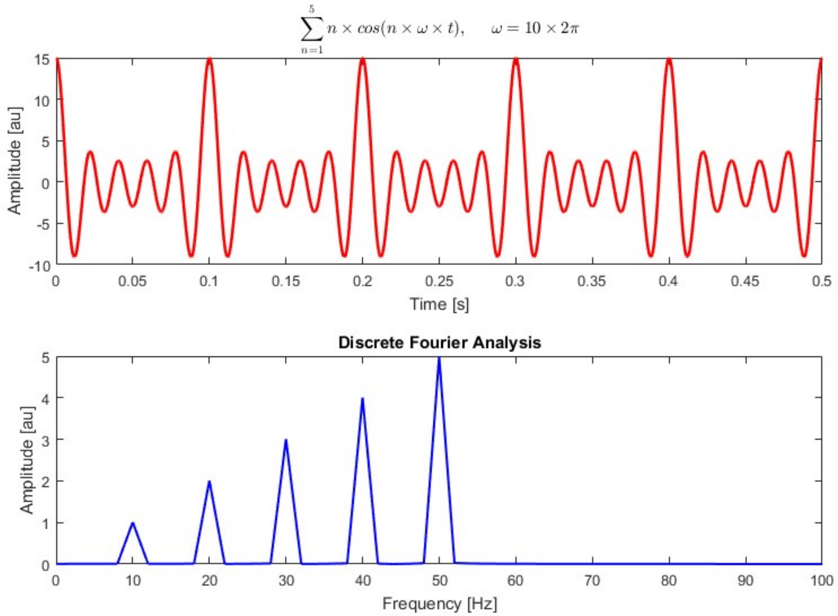
Fourier series

Fourier series (/ˈfʊrieɪ, -iər/)[1] is a periodic function composed of harmonically related sinusoids, combined by a weighted summation. With appropriate weights, one cycle (or period) of the summation can be made to approximate an arbitrary function in that interval (or the entire function if it too is periodic). As such, the summation is a synthesis of another function.

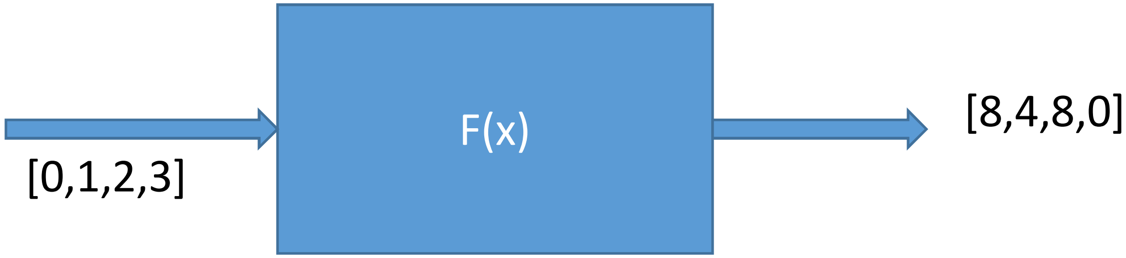
Jean-Baptiste Joseph Fourier



Discrete Fourier transform

Forward Discrete Fourier transform

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k \cdot e^{\frac{-i2\pi k}{N}}$$



What is the function in this box ?

Discrete Fourier transform

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k \cdot e^{\frac{-i2\pi kn}{N}}$$

n	X	
[0,1,2,3]	[8,4,8,0]	N=4

n=0

$$x_0 = \frac{1}{4}\left(8e^{\frac{-i2\quad\times 0}{4}} + 4e^{\frac{-i2\pi 1\times 0}{4}} + 8e^{\frac{-i2\pi 2\times 0}{4}} + 0e^{\frac{-i2\pi 3\times 0}{4}}\right)$$

$$x_0 = \frac{1}{4}((8+0i) + (4+0i) + (8+0j) + (0i)) = 5+0i$$

n=1

$$x_1 = \frac{1}{4}\left(8e^{\frac{-i2\pi\quad\times 1}{4}} + 4e^{\frac{-i2\pi 1\times 1}{4}} + 8e^{\frac{-i2\pi 2\times 1}{4}} + 0e^{\frac{-i2\pi 3\times 1}{4}}\right)$$

$$x_1 = \frac{1}{4}((8+0i) + (0-4i) + (-8+0j) + (0)) = 0-1i$$

$$\color{red}{[\mathbf{5}, -\boldsymbol{i}, \mathbf{3}, \boldsymbol{i}]}$$

n=2

$$x_2 = \frac{1}{4}\left(8e^{\frac{-i2\pi\quad\times 2}{4}} + 4e^{\frac{-i2\pi 1\times 2}{4}} + 8e^{\frac{-i2\pi 2\times 2}{4}} + 0e^{\frac{-i2\pi 3\times 2}{4}}\right)$$

$$x_2 = \frac{1}{4}((8+0i) + (-4+0i) + (8+0j) + (0)) = 3+0i$$

n=3

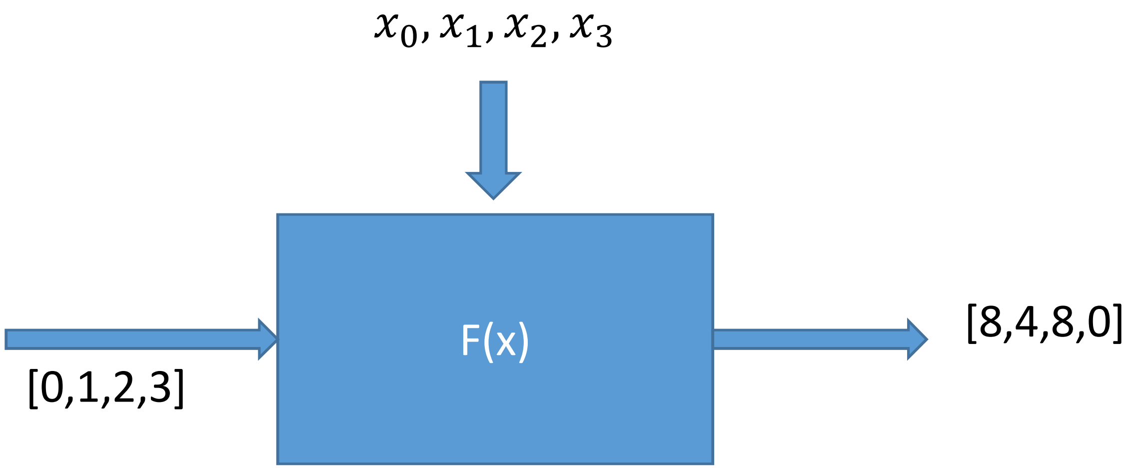
$$x_3 = \frac{1}{4}\left(8e^{\frac{-i2\pi 0\times 3}{4}} + 4e^{\frac{-i2\pi 1\times 3}{4}} + 8e^{\frac{-i2\pi 2\times 3}{4}} + 0e^{\frac{-i2\pi 3\times 3}{4}}\right)$$

$$x_3 = \frac{1}{4}((8+0i) + (0+4i) + (-8+0j) + (0)) = 0+1i$$

Discrete Fourier transform

Forward Discrete Fourier transform

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k \cdot e^{\frac{-i2\pi kn}{N}}$$



Inverse Discrete Fourier transform

$$X_n = N\sum_{k=0}^{N-1} x_k \cdot e^{\frac{i2\pi kn}{N}}$$

Discrete Fourier transform

n

[0,1,2,3]

N=4

x

$[5, -i, 3, i]$

Inverse Discrete Fourier transform

$$X_n = N \sum_{k=0}^{N-1} x_k \cdot e^{\frac{i2\pi kn}{N}}$$

n=0

$$x_0 = 4\left(5e^{\frac{i2\pi 0\times 0}{4}} \pm -ie^{\frac{i2\pi 1\times 0}{4}} + 3e^{\frac{i2\pi 2\times 0}{4}} + ie^{\frac{i2\pi 3\times 0}{4}}\right)$$

$$x_0 = 4((1.25 + 0i) + (-0.25i) + (0.75 + 0i) + (0.25i)) = 8$$

n=1

$$x_1 = 4\left(5e^{\frac{i2\pi 0\times 1}{4}} \pm -ie^{\frac{i2\pi 1\times 1}{4}} + 3e^{\frac{i2\pi 2\times 1}{4}} + ie^{\frac{i2\pi \ \times 1}{4}}\right)$$

$$x_1 = 4((1.25 + 0i) + (0.25) + (-0.75) + (0.25)) = 4$$

n=2

$$x_2 = 4\left(5e^{\frac{i2\pi 0\times 2}{4}} \pm -ie^{\frac{i2\pi \ \times 2}{4}} + 3e^{\frac{i2\pi \ \times 2}{4}} + ie^{\frac{i2\pi 3\times 2}{4}}\right)$$

$$x_2 = 4((1.25 + 0i) + (0.25i) + (0.75) + (-0.25i)) = 8$$

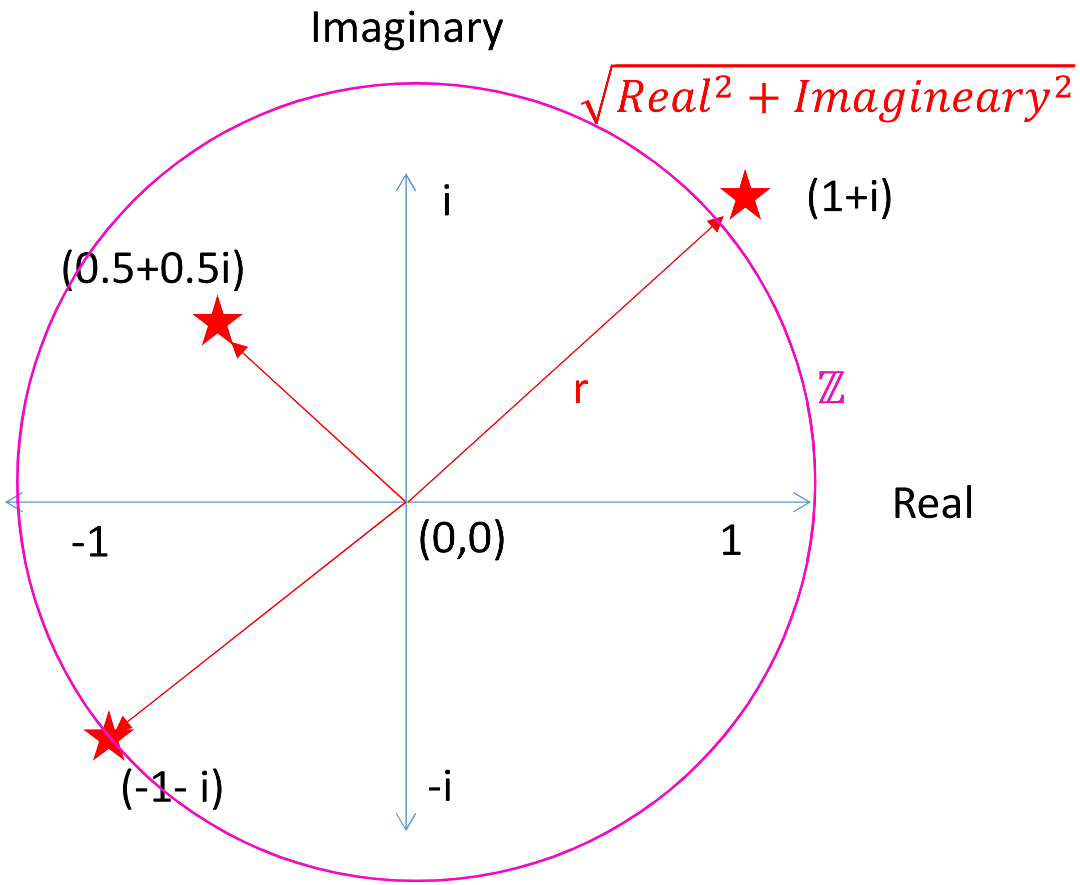
n=3

$$x_1 = 4\left(5e^{\frac{i2\pi 0\times 3}{4}} \pm -ie^{\frac{i2\pi 1\times 3}{4}} + 3e^{\frac{i2\pi 2\times 3}{4}} + ie^{\frac{i2\pi 3\times 3}{4}}\right)$$

$$x_1 = 4((1.25 + 0i) + (-0.25) + (-0.75) + (-0.25)) = 0$$

$[8, 4, 8, 0]$

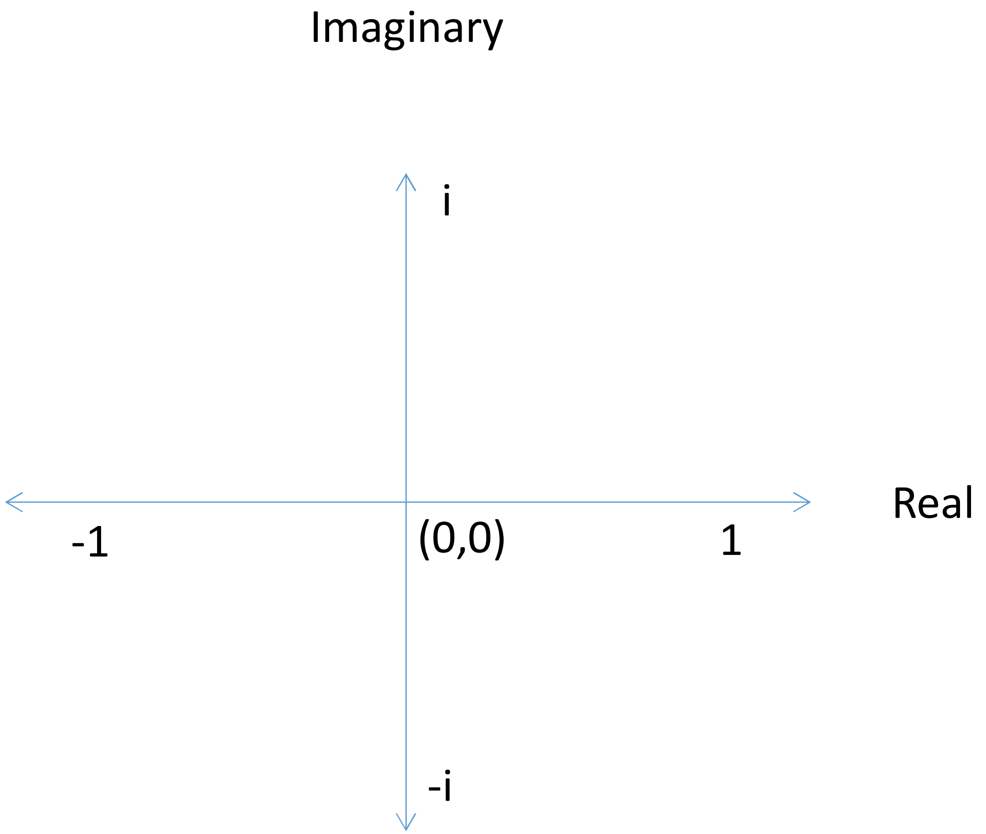
Complex Number



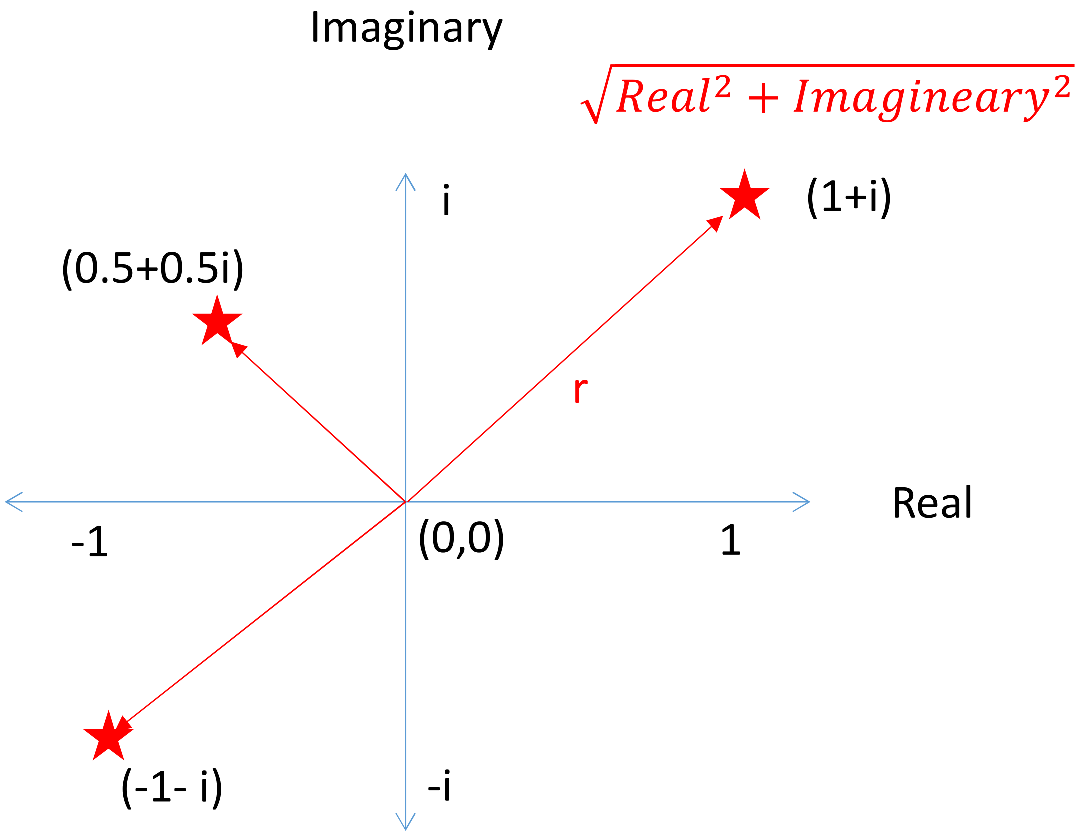
$$r = \sqrt{Real^2 + Imagineary^2} = |\mathcal{Z}|$$

$$|\mathcal{Z}| = \sqrt{[Re(\mathcal{Z})]^2 + [Im(\mathcal{Z})]^2}$$

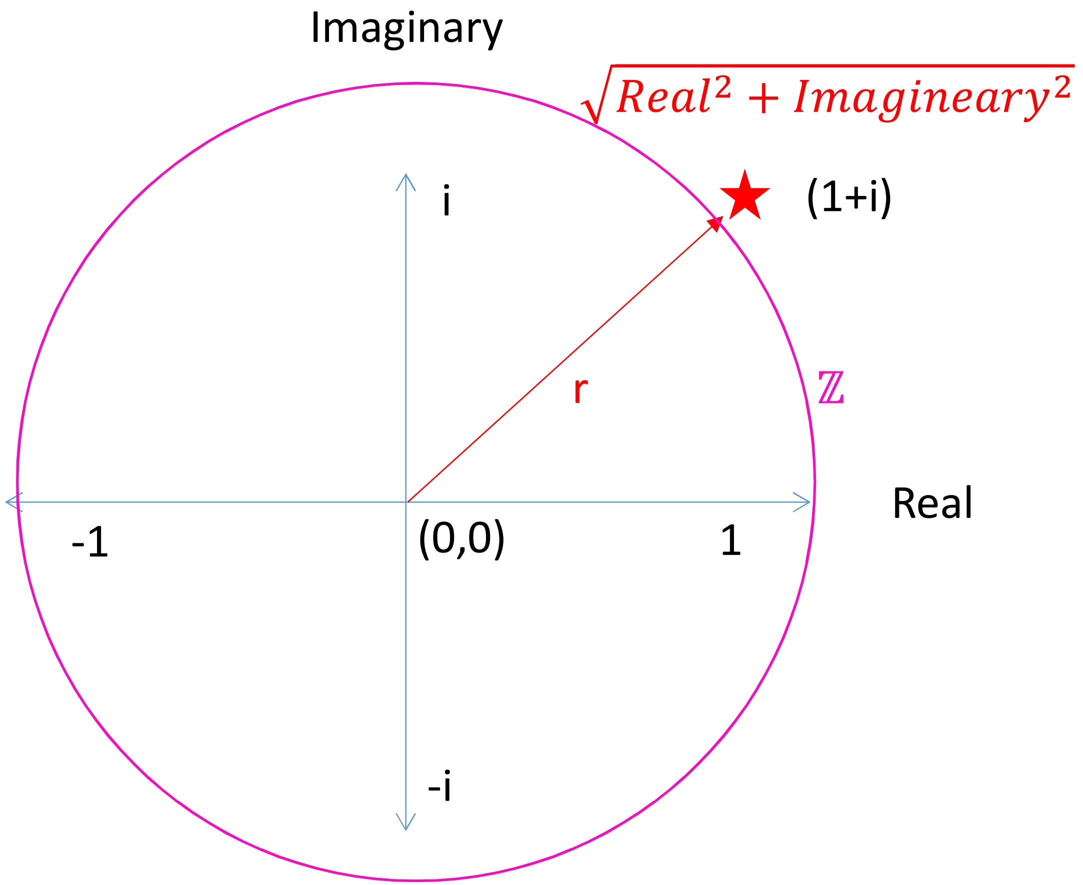
Complex Number



Complex Number



Complex Number



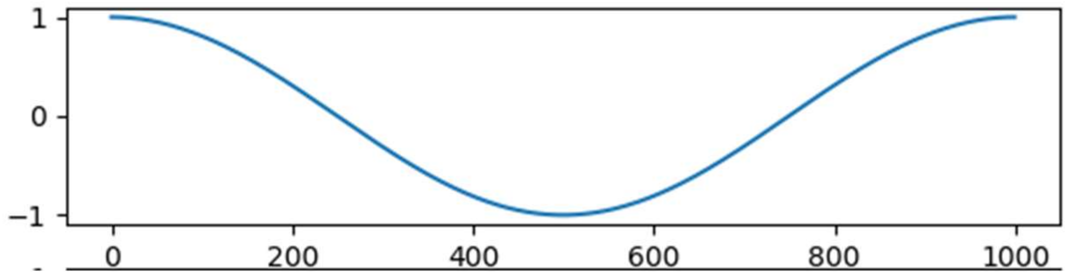
$$r = \sqrt{Real^2 + Imaginary^2} = |\mathcal{Z}|$$

$$|\mathcal{Z}| = \sqrt{[Re(\mathcal{Z})]^2 + [Im(\mathcal{Z})]^2}$$

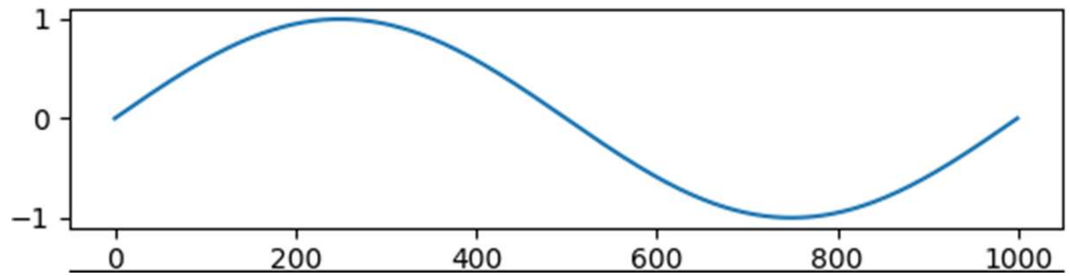
Complex Number

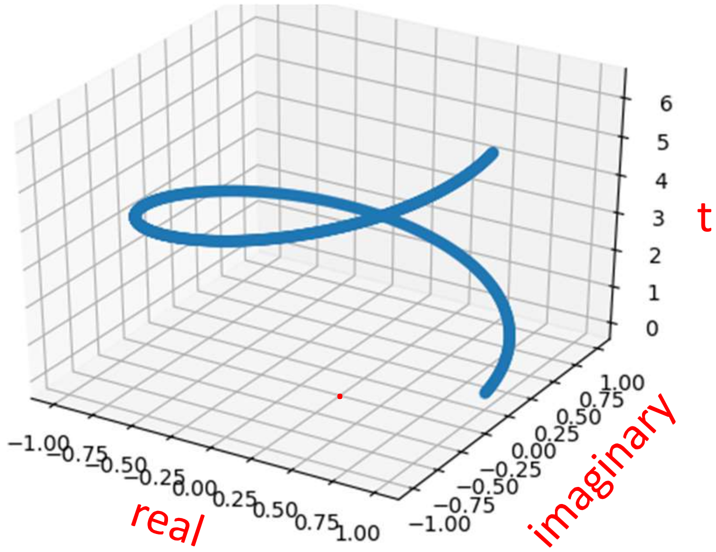
$$Y(t) = e^{it}\,;\, t = [0..2\pi]$$

Real
(cosine)



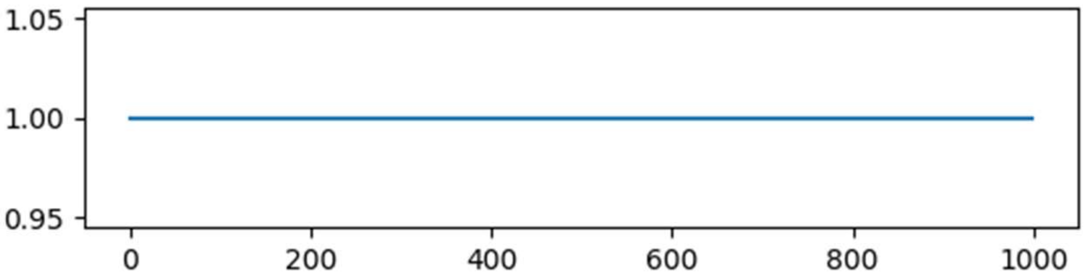
Imaginary
(sine)

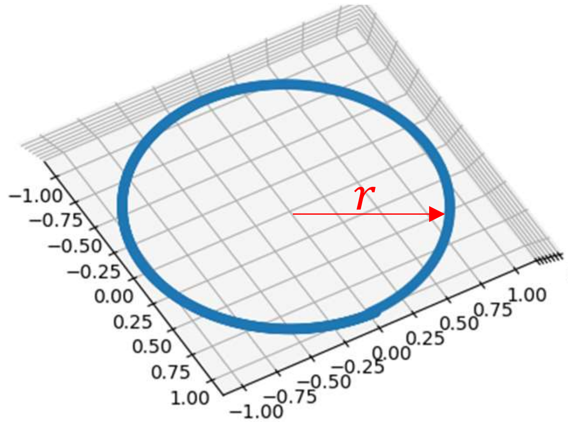




$$r = |\mathcal{Z}|$$

$|Y|$





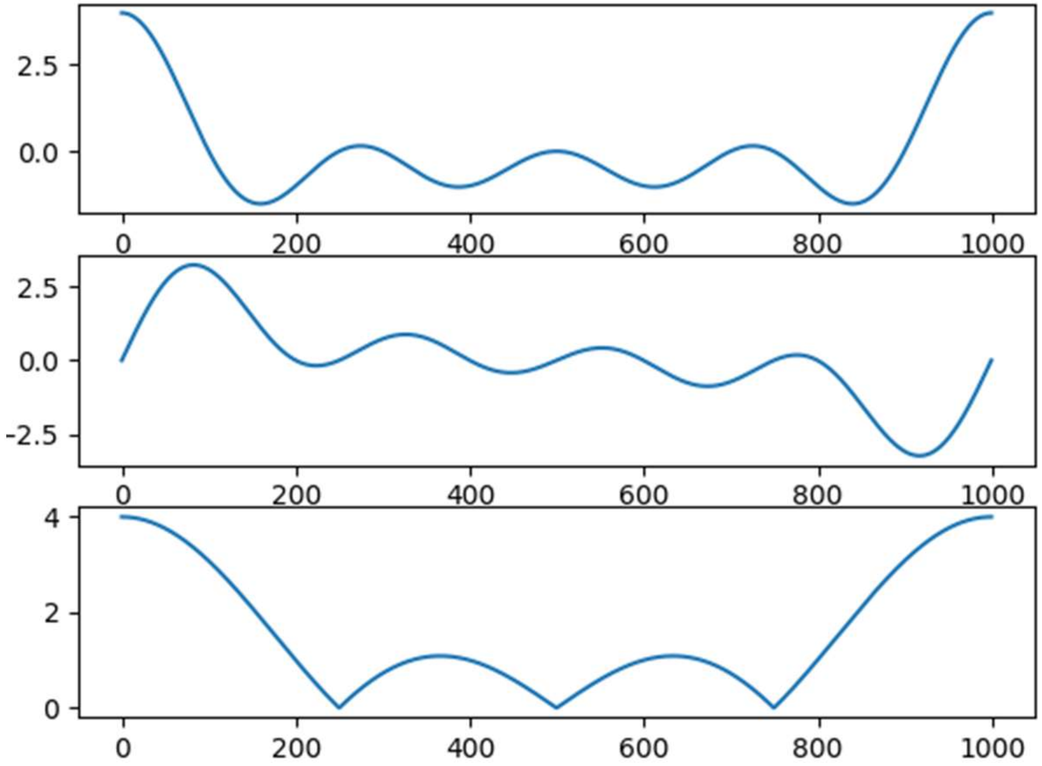
Complex Number

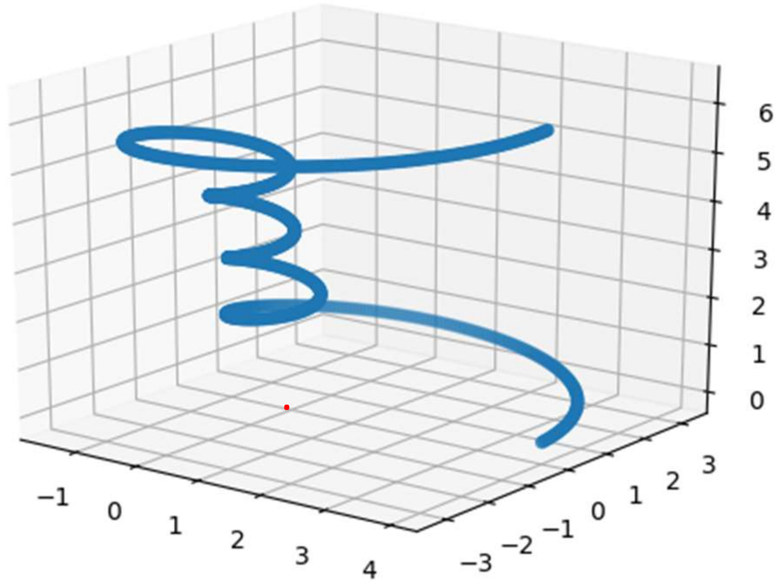
$$Y(t) = e^{it} + e^{2it} + e^{3it} + e^{4it}; t = [0..2\pi]$$

Real (cosine)

Imaginary (sine)

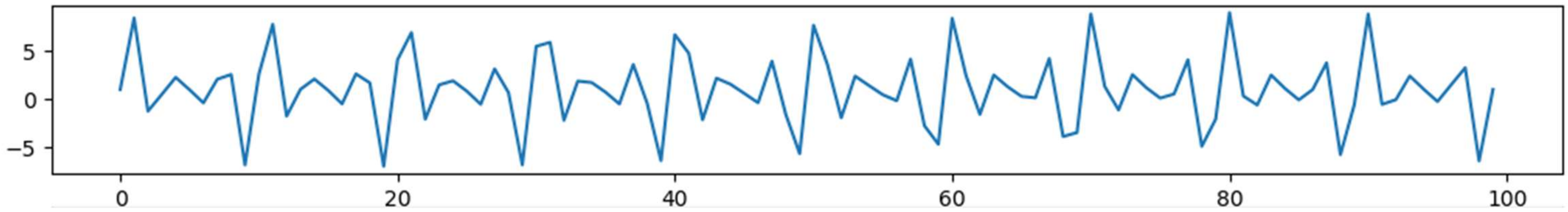
$|Y|$





Discrete Fourier transform

$$Y(t) = \sin(10t) + 2\sin(20t) + 4\sin(30t) + 2\sin(40t) + 1$$



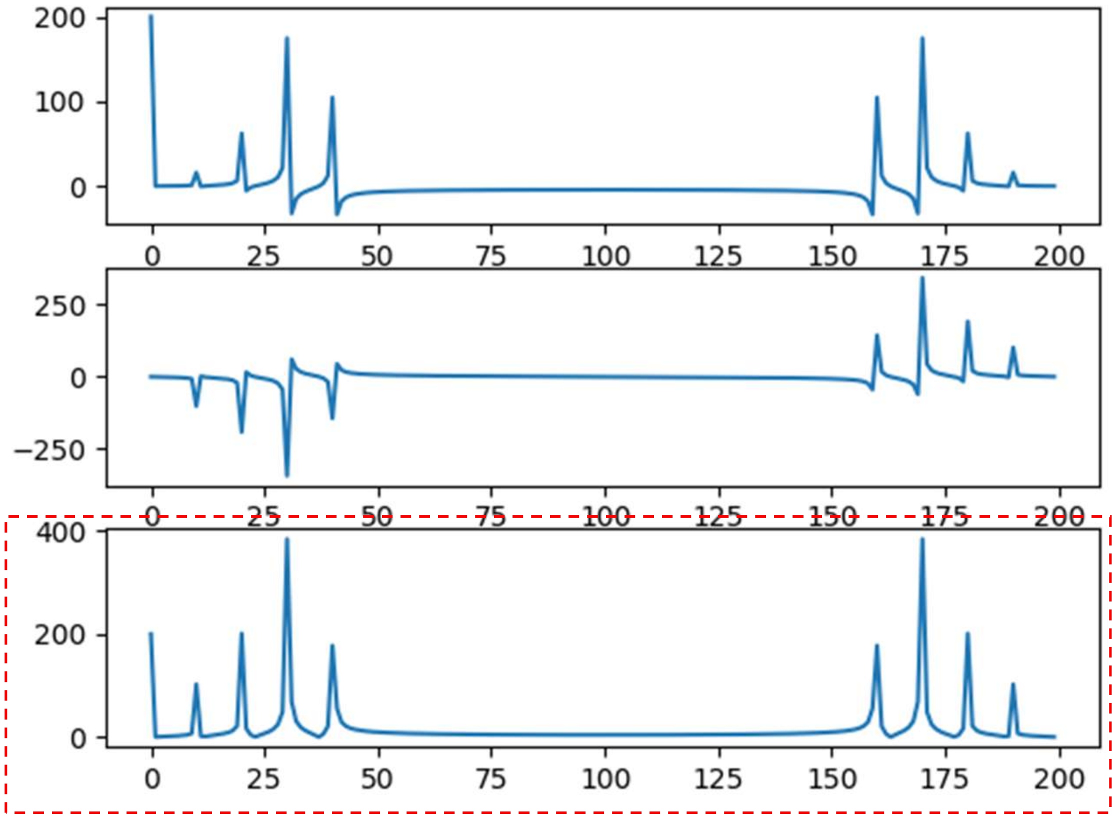
Y

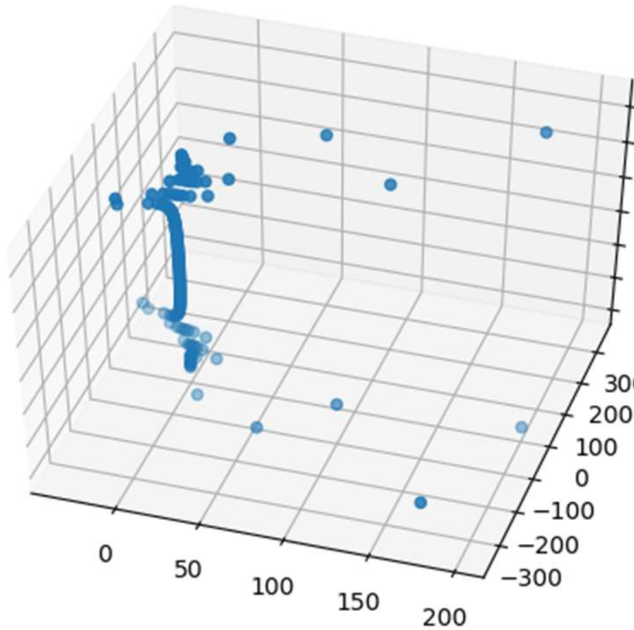
$$\mathrm{X}(t) = DFT(Y)$$

Real (cosine)

Imaginary (sine)

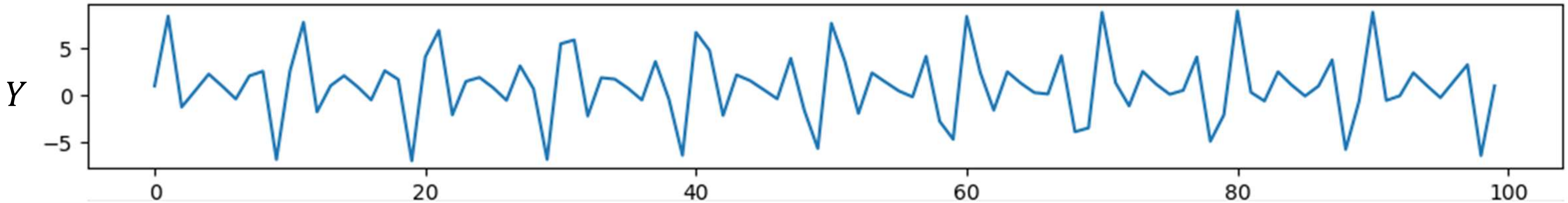
$|Y|$





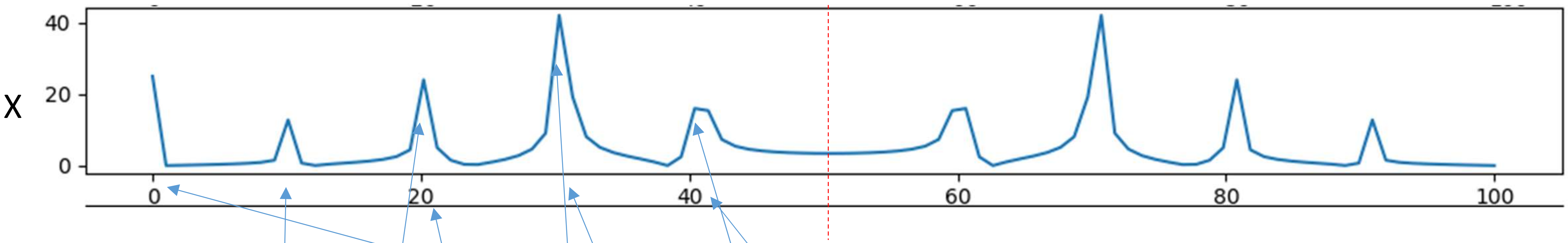
Discrete Fourier transform

$$Y(t) = \sin(10t) + 2\sin(20t) + 4\sin(30t) + 2\sin(40t) + 1$$



Envelope

$$X(t) = DFT(Y)$$

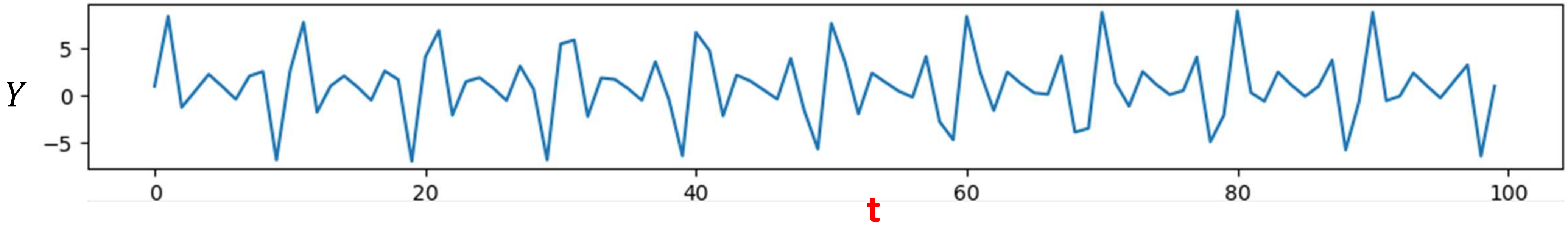


Frequency Spectrum

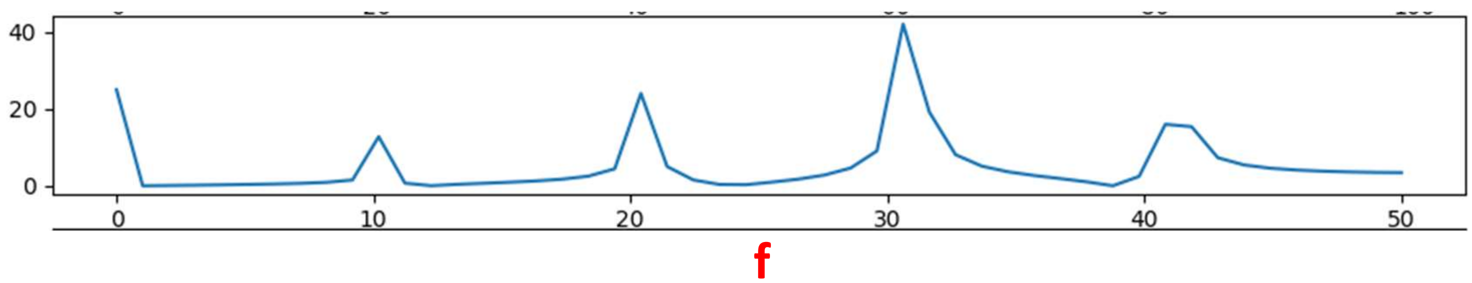
$$Y(t) = \sin(10t) + 2\sin(20t) + 4\sin(30t) + 2\sin(40t) + 1$$

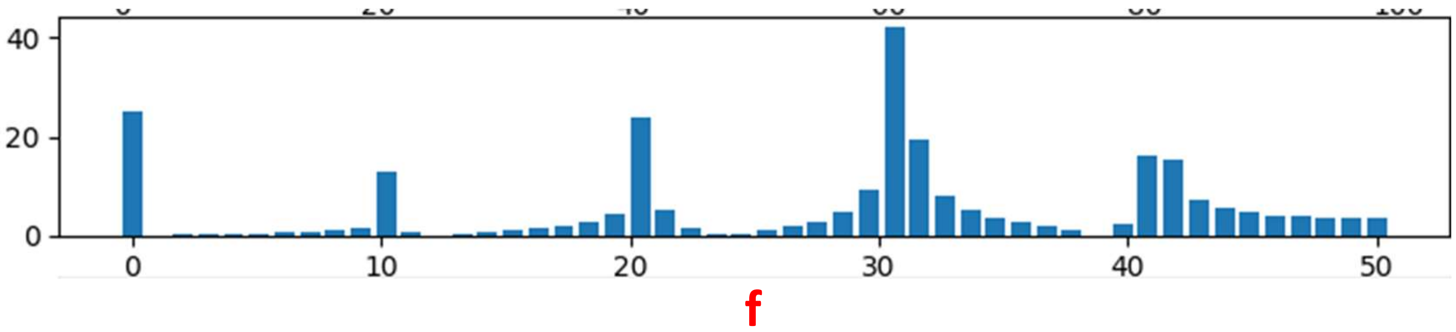
Discrete Fourier transform

$$Y(t) = \sin(10t) + 2\sin(20t) + 4\sin(30t) + 2\sin(40t) + 1$$

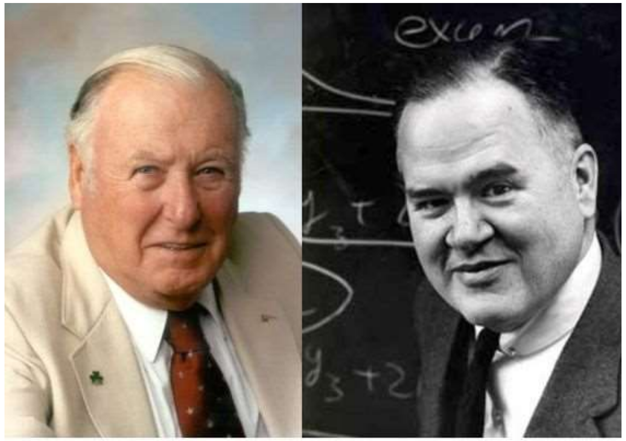


$$X(t) = DFT(Y)$$

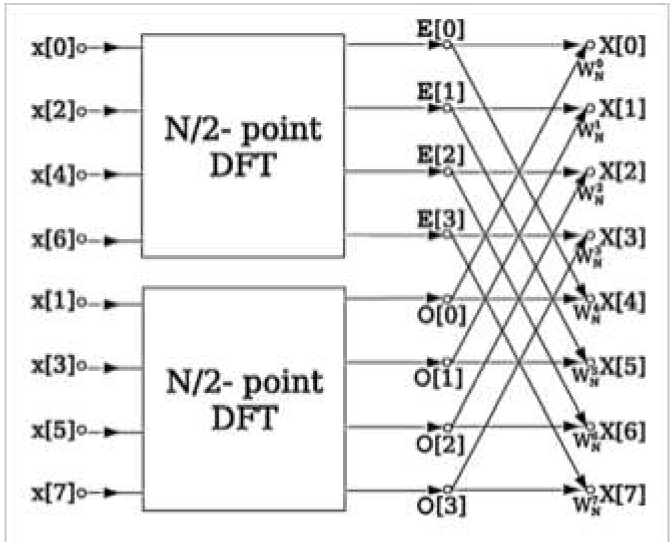




Fast Fourier transform



James William Cooley (1926 – 2016) John Wilder Tukey (1915 – 2000)



$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-i\,2\pi\,k\,n\,/\,N} \tag{1}$$

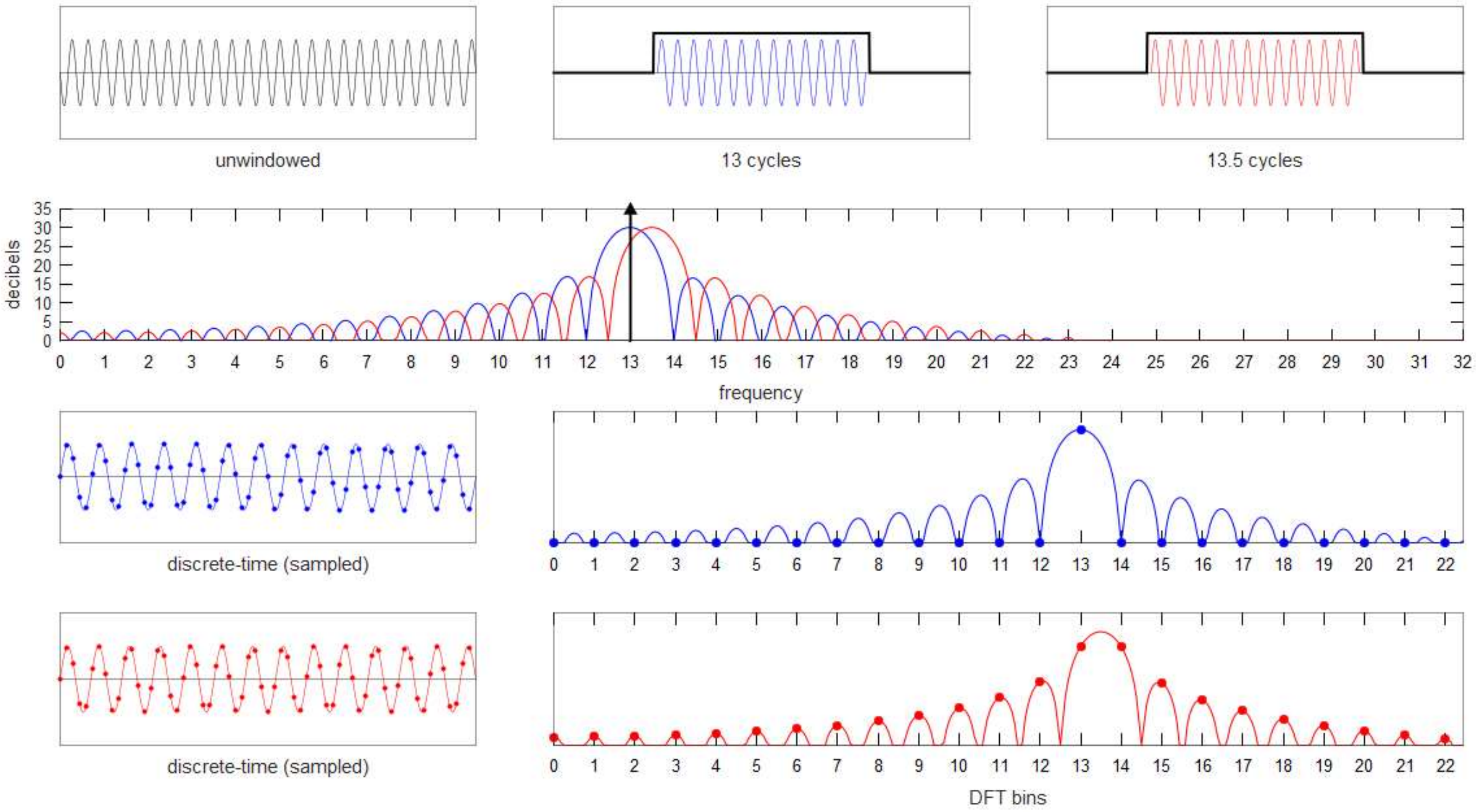
$$= \sum_{m=0}^{N/2-1} x_{2m} \cdot e^{-i\,2\pi\,k\,(2m)\,/\,N} + \sum_{m=0}^{N/2-1} x_{2m+1} \cdot e^{-i\,2\pi\,k\,(2m+1)\,/\,N} \tag{2}$$

$$= \sum_{m=0}^{N/2-1} x_{2m} \cdot e^{-i\,2\pi\,k\,m\,/\,(N/2)} + e^{-i\,2\pi\,k\,/\,N} \sum_{m=0}^{N/2-1} x_{2m+1} \cdot e^{-i\,2\pi\,k\,m\,/\,(N/2)} \tag{3}$$

$$O(N^2) \longrightarrow O(N\,log(N))$$

https://jakevdp.github.io/blog/2013/08/28/understanding-the-fft/

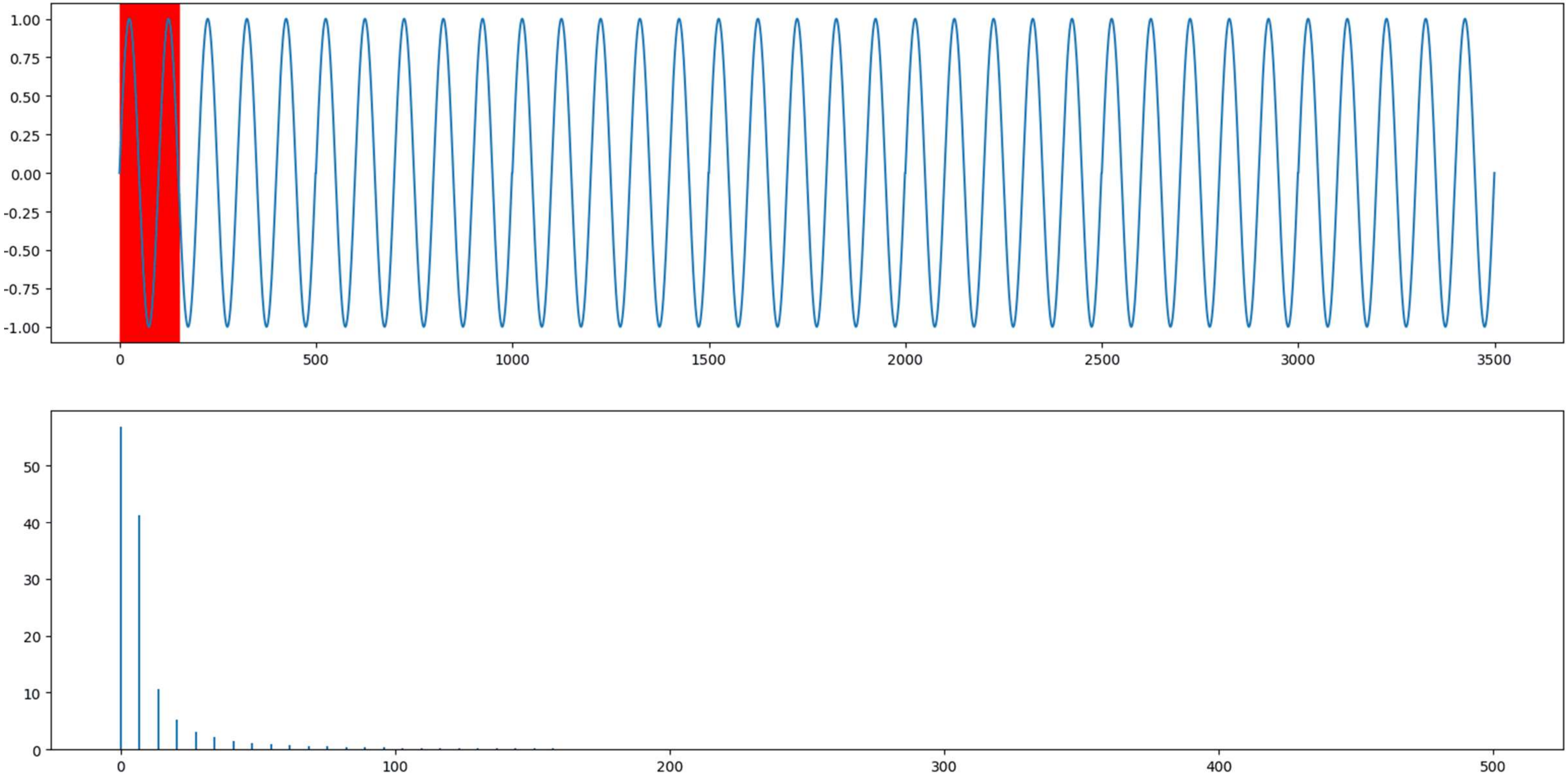
Spectral leakage caused by “windowing”



https://en.wikipedia.org/wiki/Spectral_leakage

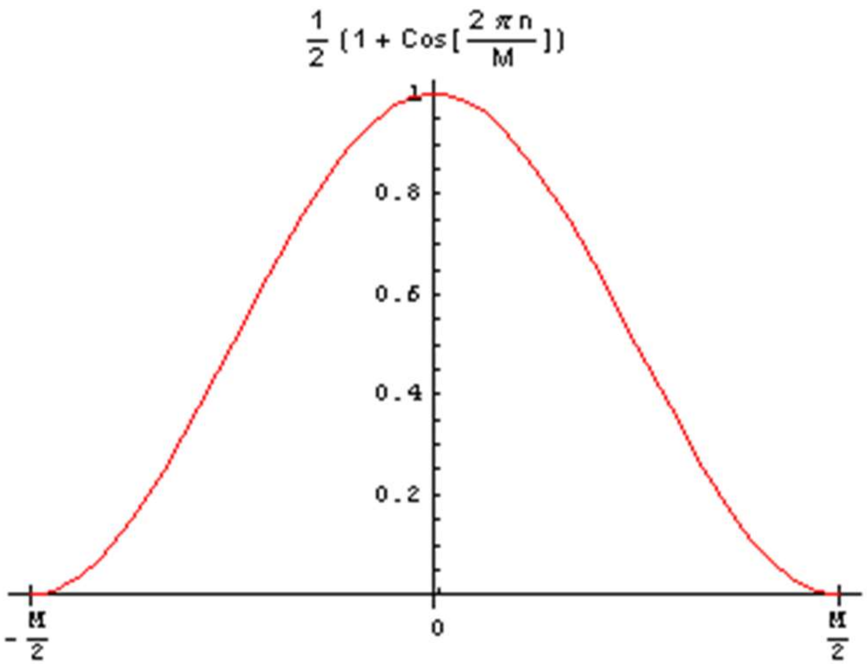
Spectral leakage

Online processing window



Window function

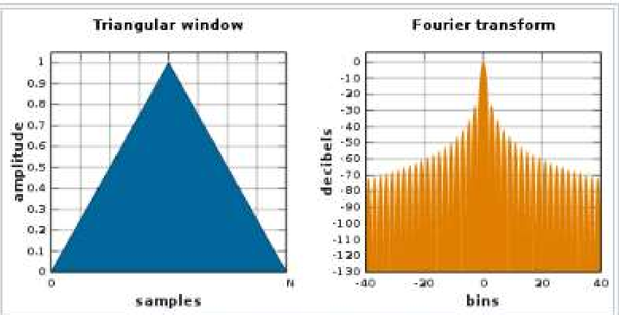
In signal processing and statistics, a **window function** (also known as an **apodization function** or **tapering function**[1]) is a mathematical function that is zero-valued outside of some chosen interval



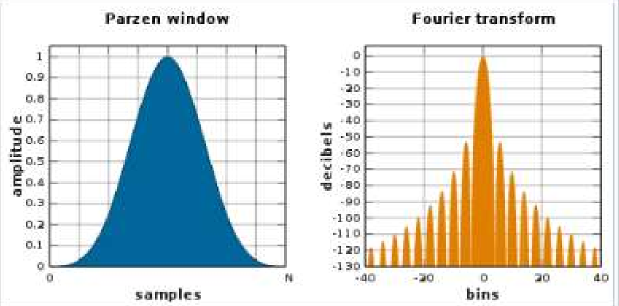
Hann window

https://en.wikipedia.org/wiki/Window_function

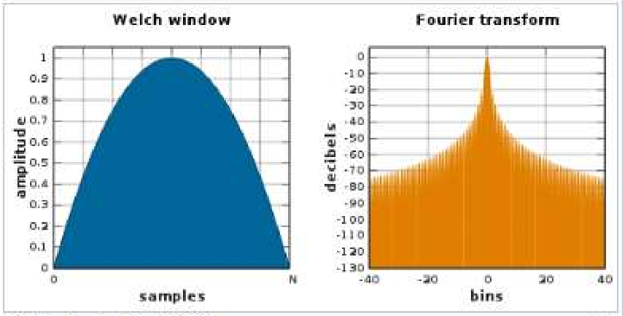
Window function



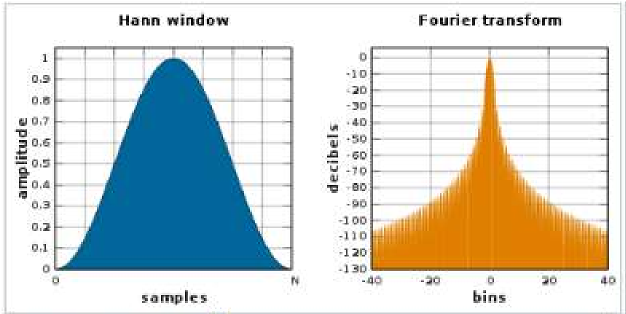
Triangular window (with $L=N$) or equivalently the Bartlett window; $B = 1.3333$.[14]



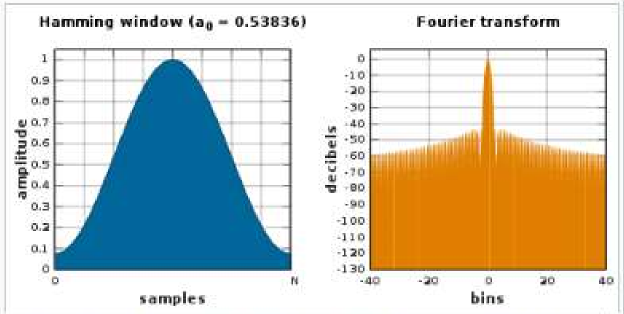
Parzen window; $B = 1.92$.[17]



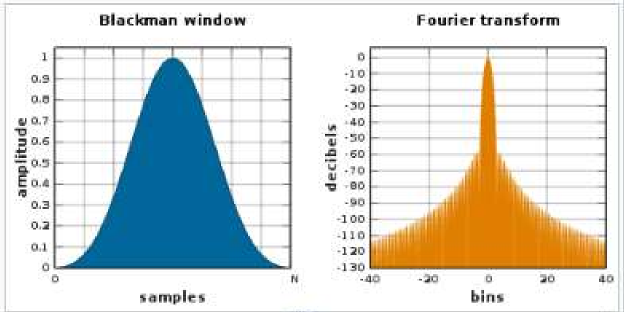
Welch window; $B = 1.20$.[17]



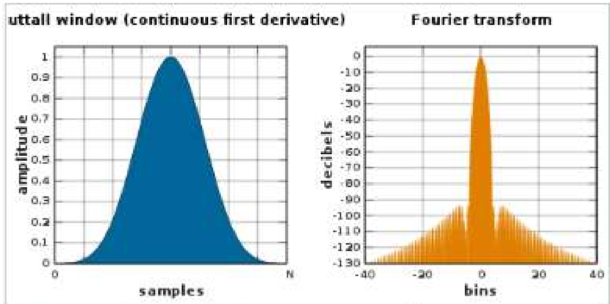
Hann window; $B = 1.5000$.[14]



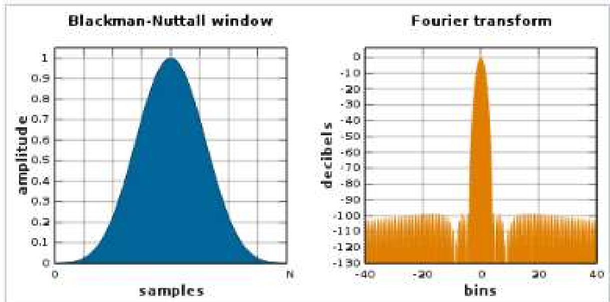
Hamming window, $a_0 = 0.53836$ and $a_1 = 0.46164$; $B = 1.37$. The original Hamming window would have $a_0 = 0.54$ and $a_1 = 0.46$; $B = 1.3628$.[14]



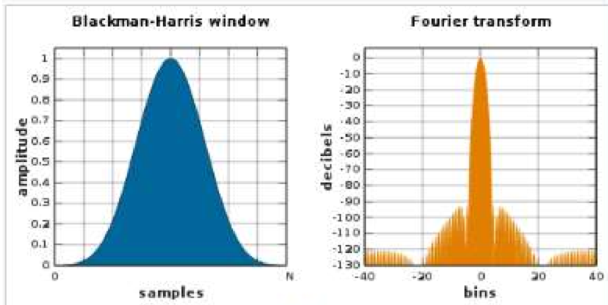
Blackman window; $\alpha = 0.16$; $B = 1.73$.[17]



Nuttall window, continuous first derivative; $B = 2.0212$.[14]



Blackman–Nuttall window; $B = 1.9761$.[14]

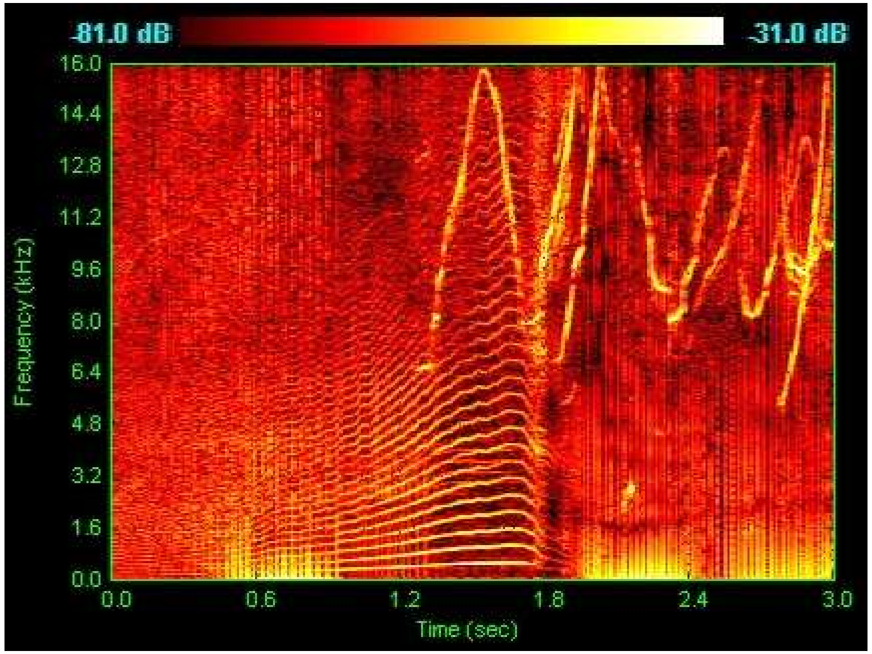


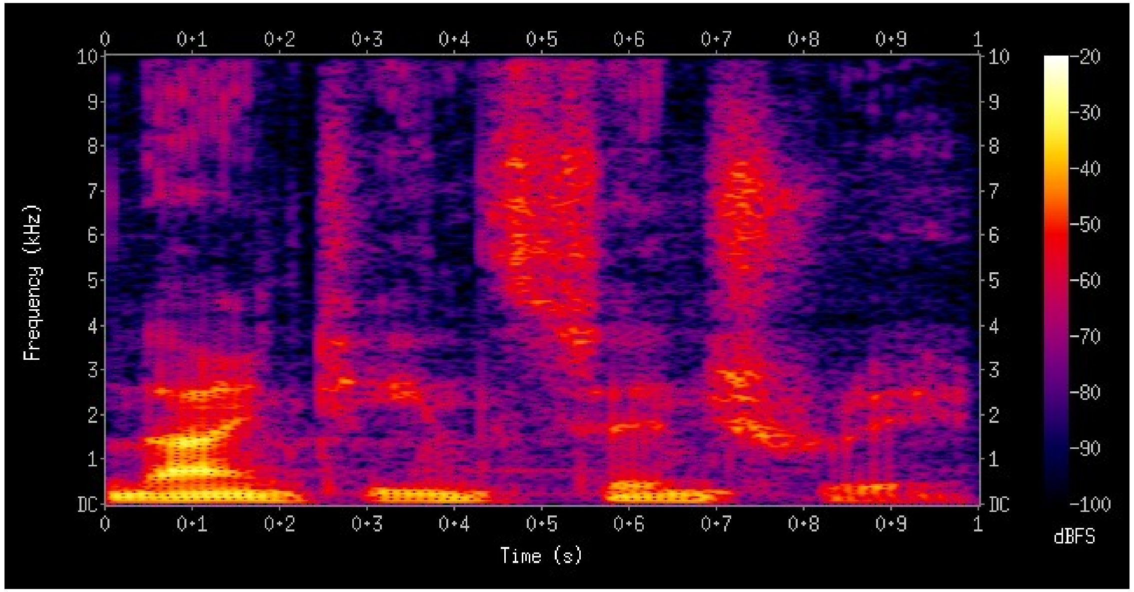
Blackman–Harris window; $B = 2.0044$.[14]

https://en.wikipedia.org/wiki/Window_function

Spectrogram

A **spectrogram** is a visual representation of the spectrum of frequencies of a signal as it varies with time. When applied to an audio signal, spectrograms are sometimes called **sonographs**, **voiceprints**, or **voicegrams**. When the data is represented in a 3D plot they may be called **waterfalls**.





https://en.wikipedia.org/wiki/Spectrogram

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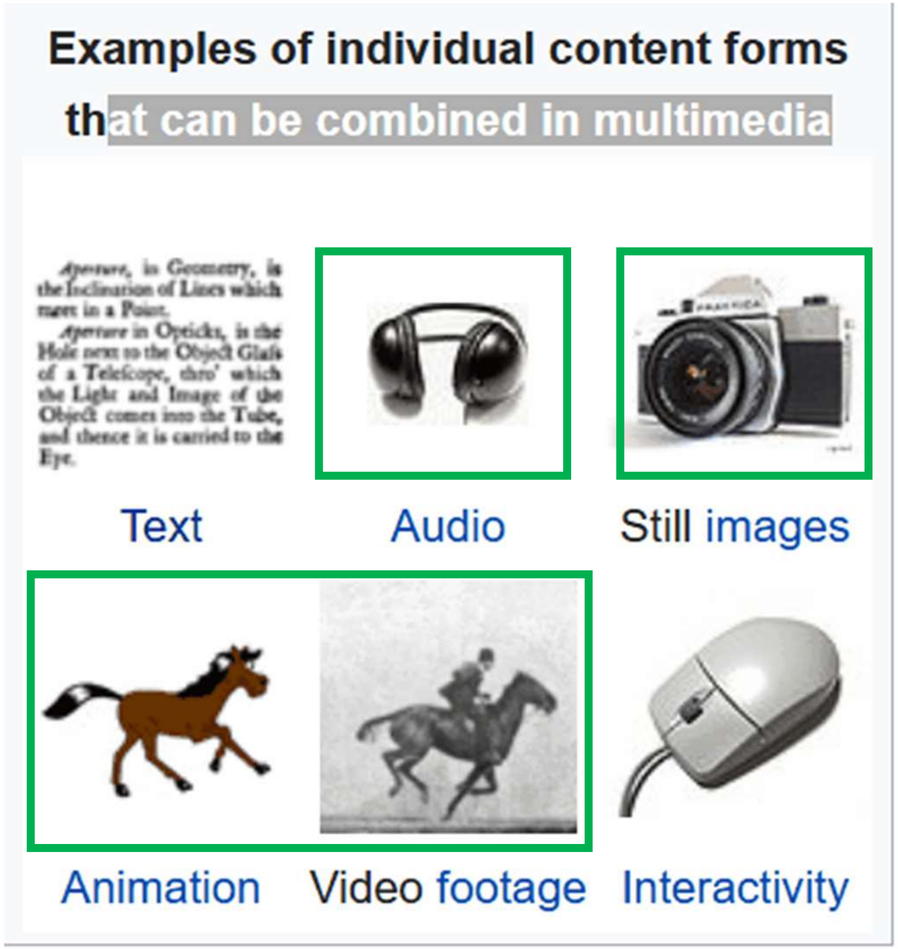

- Image Processing
- Video Processing
- Audio Processing

python

OpenCV

NumPy

matplotlib

pillow

SciPy.org