DTI 516 Multimedia Processing

Chapter: 4

Image Filters

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Image Filter

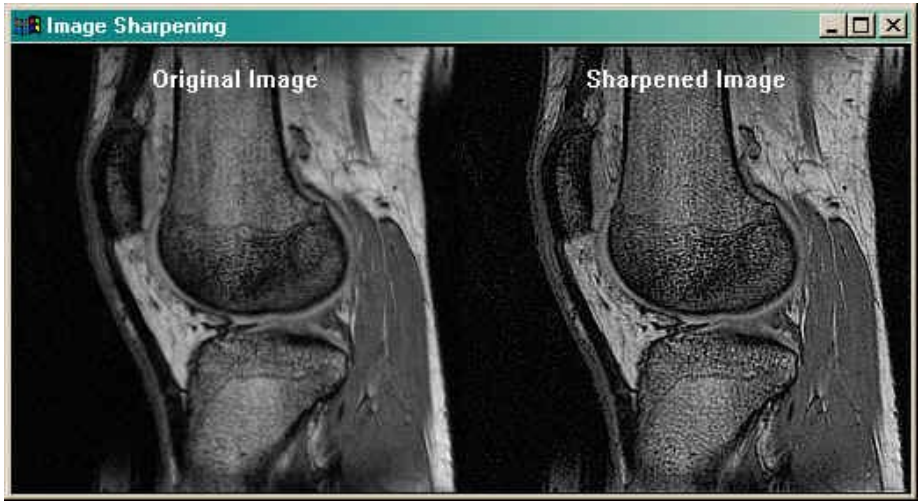
Image Courtesy of MIT

Original Image

Motion Blurred Image

Blurred Image

Sharpened Image



Spatial convolution

$$\mathbf{F}(x) * \boldsymbol{G}(x) = \int_{-\infty}^{\infty} \boldsymbol{F}(\tau) \cdot \boldsymbol{G}(x - \tau) d\tau$$

Where

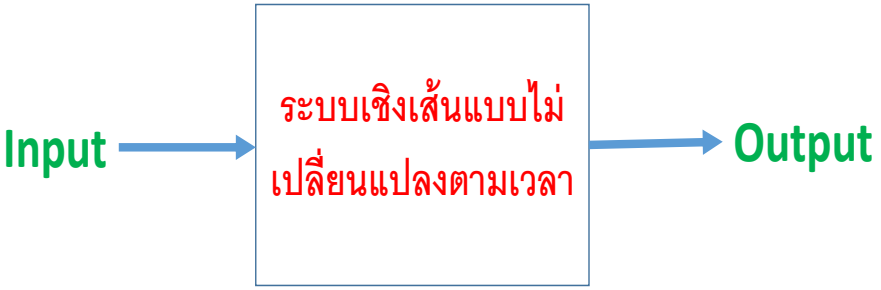
$*$ means convolution

$\boldsymbol{F}$ is image

$\boldsymbol{G}$ is convolution kernel

ระบบเชิงเส้นที่ไม่เปลี่ยนแปลงตามเวลา

ระบบเชิงเส้นที่ไม่เปลี่ยนแปลงตามเวลา(Linear time-invariant: LTI) คือระบบการประมวลผลที่มีสัญญาณขาเข้า (input) และสัญญาณขาออก (output) มีคุณสมบัติ 2 ประการคือ เป็นเชิงเส้น และไม่เปลี่ยนแปลงตามเวลา



เป็นเชิงเส้น

หากระบบ $x_1(t)$ ทำให้เกิด $y_1(t)$ และ $x_2(t)$ ทำให้เกิด $y_2(t)$ แล้ว

$a_1x_1(t) + a_2x_2(t)$ จะได้ $a_1y_1(t) + a_2y_2(t)$; เมื่อ a_1 และ a_2 เป็นจำนวนจริง

ไม่เปลี่ยนแปลงตามเวลา

หากระบบ $x_1(t)$ ทำให้เกิด $y_1(t)$ แล้ว $x_1(t-T)$ ทำให้เกิด $y_1(t-T)$ ด้วยเช่นกัน

โดยที่ T คือเวลาที่ถูกหน่วง

Spatial convolution

F

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

G

0.25	0.25
0.25	0.25

$$\mathbf{I}(x) = \mathbf{F}(x) * \boldsymbol{G}(x)$$

Spatial convolution

F

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

G

0.25	0.25
0.25	0.25

I

Spatial convolution

F

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

G

0.25	0.25
0.25	0.25

I

x							

$\boldsymbol{F(\tau)}$

1	2
9	10

$\bullet$

G

0.25	0.25
0.25	0.25

$$I(1,1) = (1 \times 0.25) + (2 \times 0.25) + (9 \times 0.25) + (10 \times 0.25)$$

$$I(1,1) = 5.5$$

Spatial convolution

F

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

G

0.25	0.25
0.25	0.25

I

5.5							

$\boldsymbol{F(\tau)}$

1	2
9	10

$\bullet$

G

0.25	0.25
0.25	0.25

$$I(1,1) = (1 \times 0.25) + (2 \times 0.25) + (9 \times 0.25) + (10 \times 0.25)$$

$$I(1,1) = 5.5$$

Spatial convolution

F

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

G

0.25	0.25
0.25	0.25

I

5.5	x						

$\boldsymbol{F(\tau)}$

2	3
10	11

$\bullet$

G

0.25	0.25
0.25	0.25

$$I(1,1) = (2 \times 0.25) + (3 \times 0.25) + (10 \times 0.25) + (11 \times 0.25)$$

$$I(1,1) = 6.5$$

Spatial convolution

F

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

G

0.25	0.25
0.25	0.25

I

5.5	6.5						

$\boldsymbol{F(\tau)}$

2	3
10	11

$\bullet$

G

0.25	0.25
0.25	0.25

$$I(1,1) = (2 \times 0.25) + (3 \times 0.25) + (10 \times 0.25) + (11 \times 0.25)$$

$$I(1,1) = 6.5$$

Spatial convolution

F

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

G

0.25	0.25
0.25	0.25

I

5.5	**6.5**	**7.5**	**8.5**	**9.5**	**10.5**	**11.5**	

Spatial convolution

F

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

G

0.25	0.25
0.25	0.25

I

5.5	6.5	7.5	8.5	9.5	10.5	11.5	
x							

Spatial convolution

F

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

G

0.25	0.25
0.25	0.25

I

5.5	**6.5**	**7.5**	**8.5**	**9.5**	**10.5**	**11.5**	
13.5	**14.5**	**15.5**	**16.5**	**17.5**	**18.5**	**19.5**	

Spatial convolution

F

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

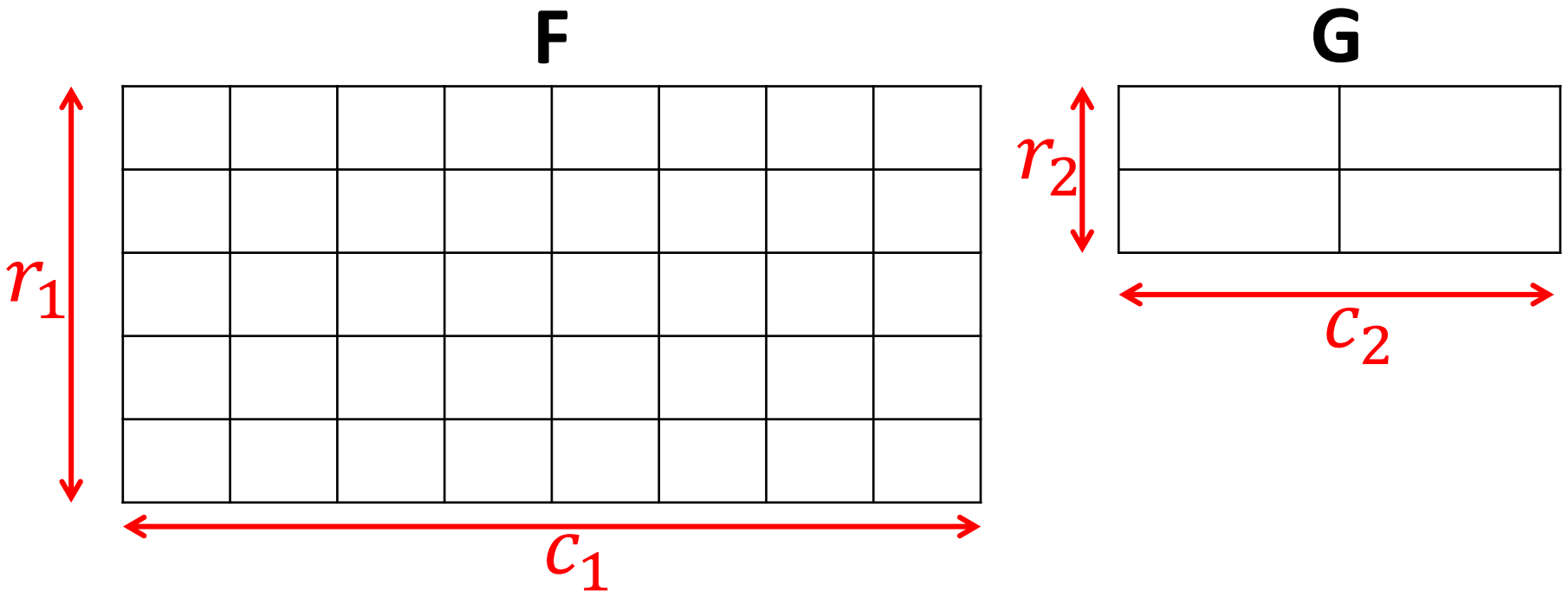
G

0.25	0.25
0.25	0.25

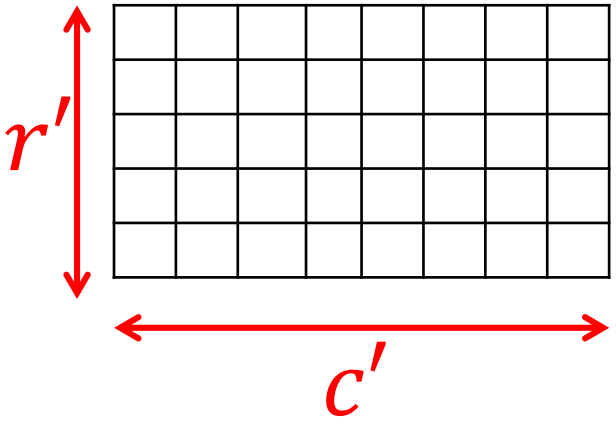
I

5.5	6.5	7.5	8.5	9.5	10.5	11.5
13.5	14.5	15.5	16.5	17.5	18.5	19.5
21.5	22.5	23.5	24.5	25.5	26.5	27.5
29.5	30.5	31.5	32.5	33.5	34.5	35.5

Spatial convolution



The final size



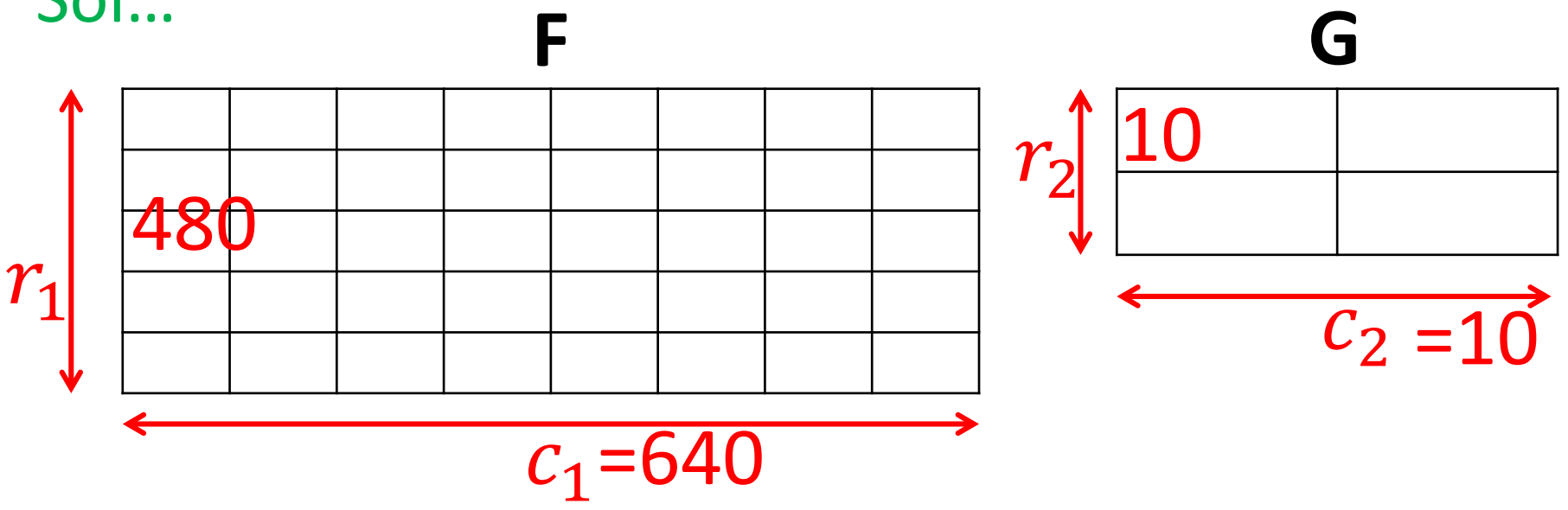
$$r' = r_1 - (r_2 - 1)$$

$$c' = c_1 - (c_2 - 1)$$

Question !

The original width and height of image **I** are 640 and 480 pixels, please determine the final dimension of this image when convolution with kernel size 10x10?

Sol…



$$r' = r_1 - (r_2 - 1,)$$

$$r' = 480 - (10 - 1)$$

$$r' = 471$$

$$c' = c_1 - (c_2 - 1)$$

$$c' = 640 - (10 - 1)$$

$$c' = 631$$

Spatial convolution (Zero pad)

F

1	2	3	4	5	6	7	8	0
9	10	11	12	13	14	15	16	0
17	18	19	20	21	22	23	24	0
25	26	27	28	29	30	31	32	0
33	34	35	36	37	38	39	40	0
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0

G

0.25	0.25
0.25	0.25

I

5.5	**6.5**	**7.5**	**8.5**	**9.5**	**10.5**	**11.5**	**6**
13.5	**14.5**	**15.5**	**16.5**	**17.5**	**18.5**	**19.5**	10
21.5	**22.5**	**23.5**	**24.5**	**25.5**	**26.5**	**27.5**	14
29.5	**30.5**	**31.5**	**32.5**	**33.5**	**34.5**	**35.5**	18
16.7	17.2	17.7	18.2	18.7	19.2	19.7	10

Spatial convolution (Replicate)

F

1	2	3	4	5	6	7	8	8
9	10	11	12	13	14	15	16	16
17	18	19	20	21	22	23	24	24
25	26	27	28	29	30	31	32	32
33	34	35	36	37	38	39	40	40
33	**34**	**35**	**36**	**37**	**38**	**39**	**40**	40

G

0.25	0.25
0.25	0.25

I

5.5	**6.5**	**7.5**	**8.5**	**9.5**	**10.5**	**11.5**	**12**
13.5	**14.5**	**15.5**	**16.5**	**17.5**	**18.5**	**19.5**	20
21.5	**22.5**	**23.5**	**24.5**	**25.5**	**26.5**	**27.5**	28
29.5	**30.5**	**31.5**	**32.5**	**33.5**	**34.5**	**35.5**	36
33.5	34.5	35.5	36.5	37.5	38.8	39.5	40

Question !

207	24	40	36	167
230	71	247	107	9
32	139	244	233	216
232	244	123	202	238
161	246	204	244	173

I

Image **I** is represented as **double precision**.

Please determine the **I ∗ M** ?

1	0	1
0	-4	0
1	0	1

M

Sol..

239	**-556**	**239**
276	**-352**	**-315**
-335	**370**	**29**

I ∗ M

Quitz : คำนวณ F * G โดยไม่ต้อง pad

F

1	2	3	4
9	10	11	12

G

0.5	0.5

$$\mathbf{I} = \mathbf{F} * \boldsymbol{G}$$

I

1.5	2.5	3.5
9.5	10.5	11.5

Quitz : คำนวณ F * G โดยไม่ต้อง pad

F

1	2	3	4
9	10	11	12

G

0.5
0.5

$$\mathbf{I} = \mathbf{F} * \boldsymbol{G}$$

I

5	6	7	8

Image Filter

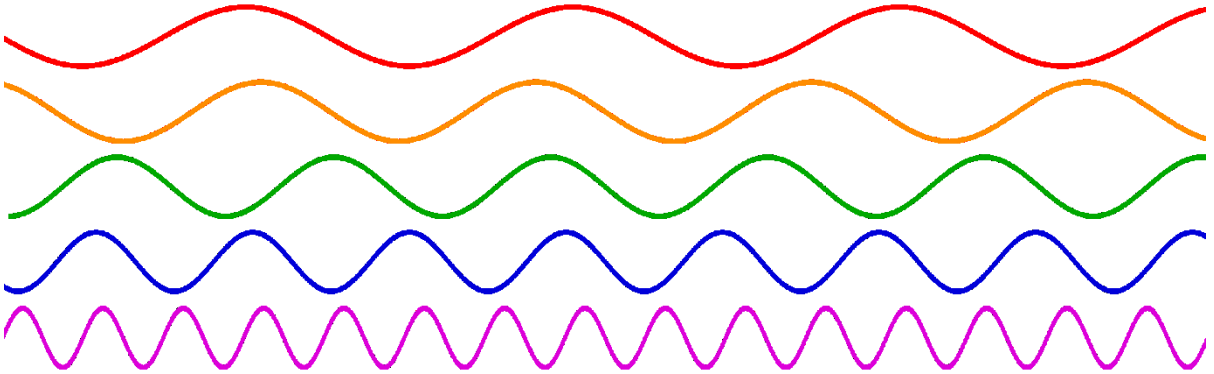
Frequency Filter

- Low-pass filter
- High-pass filter
- Band-pass filter
- Band-stop filter

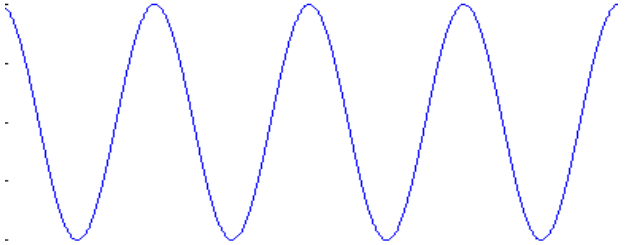
Frequency

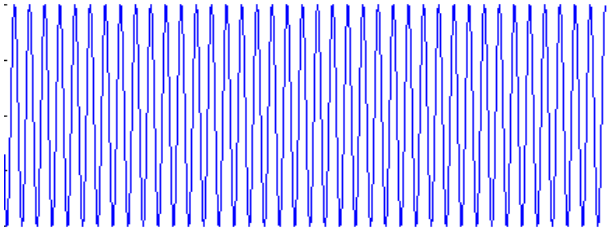
Frequency is the number of occurrences of a repeating event per unit time.

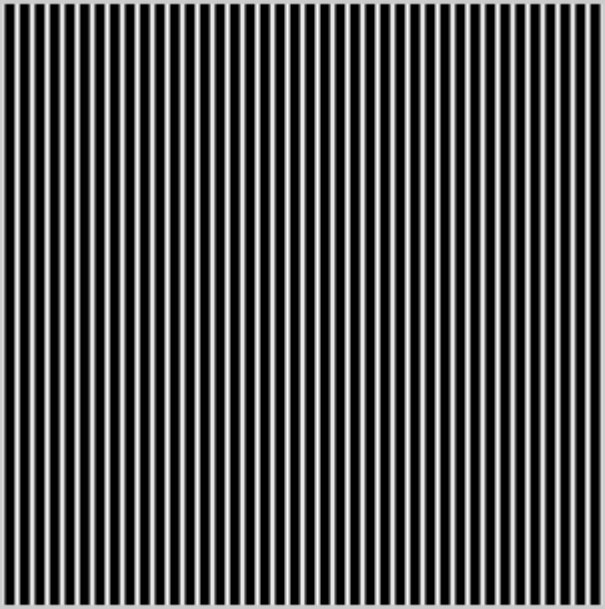
- f = 0.5 Hz
  T = 2.0 s
- f = 1.0 Hz
  T = 1.0 s
- f = 2.0 Hz
  T = 0.5 s



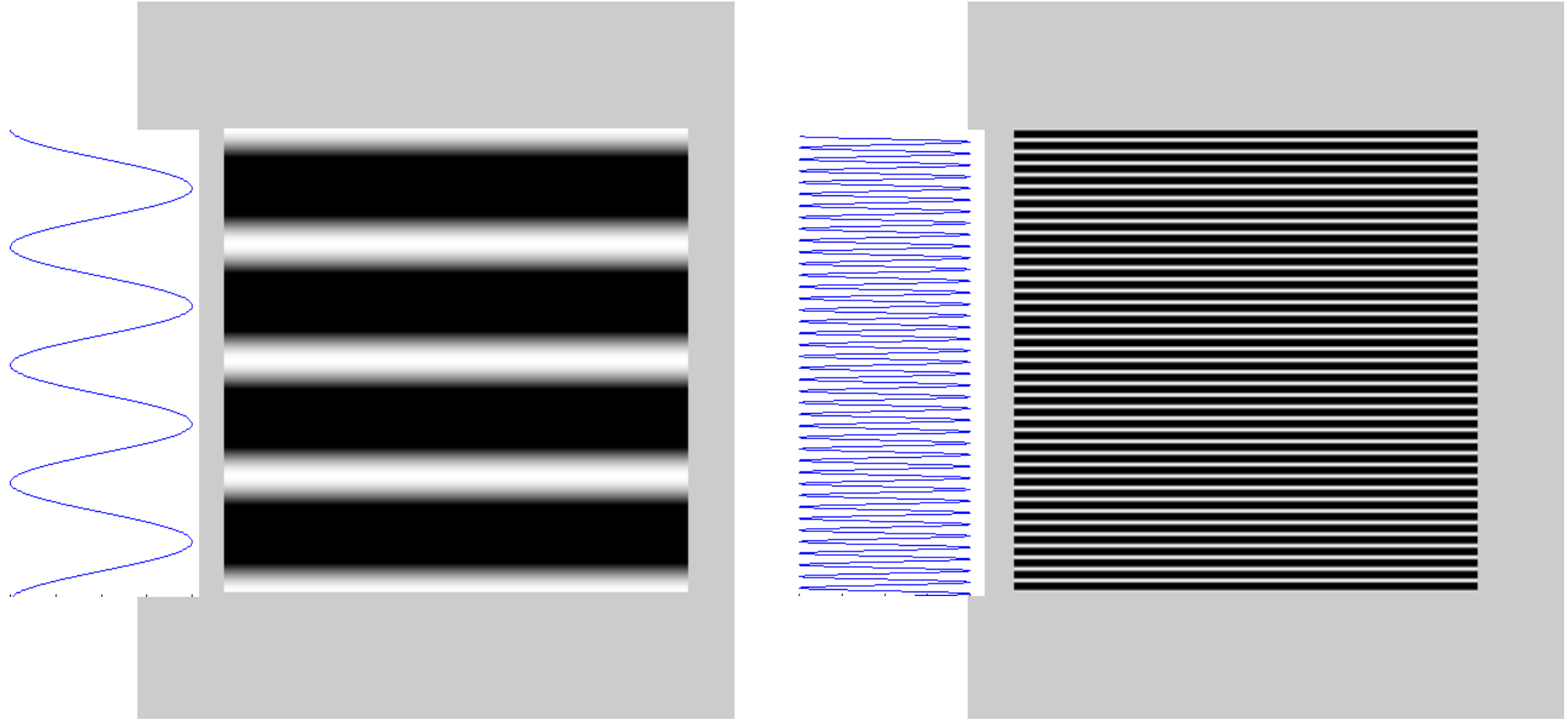
Frequency



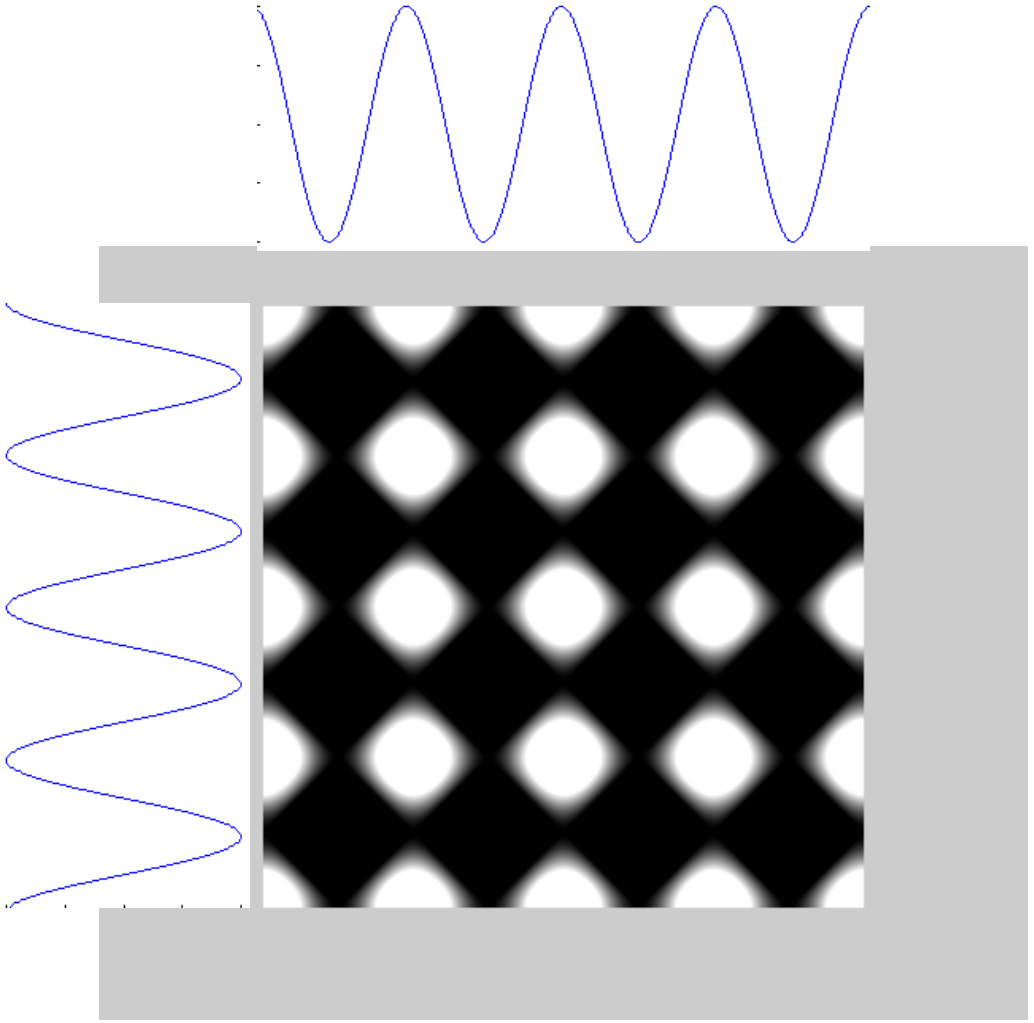


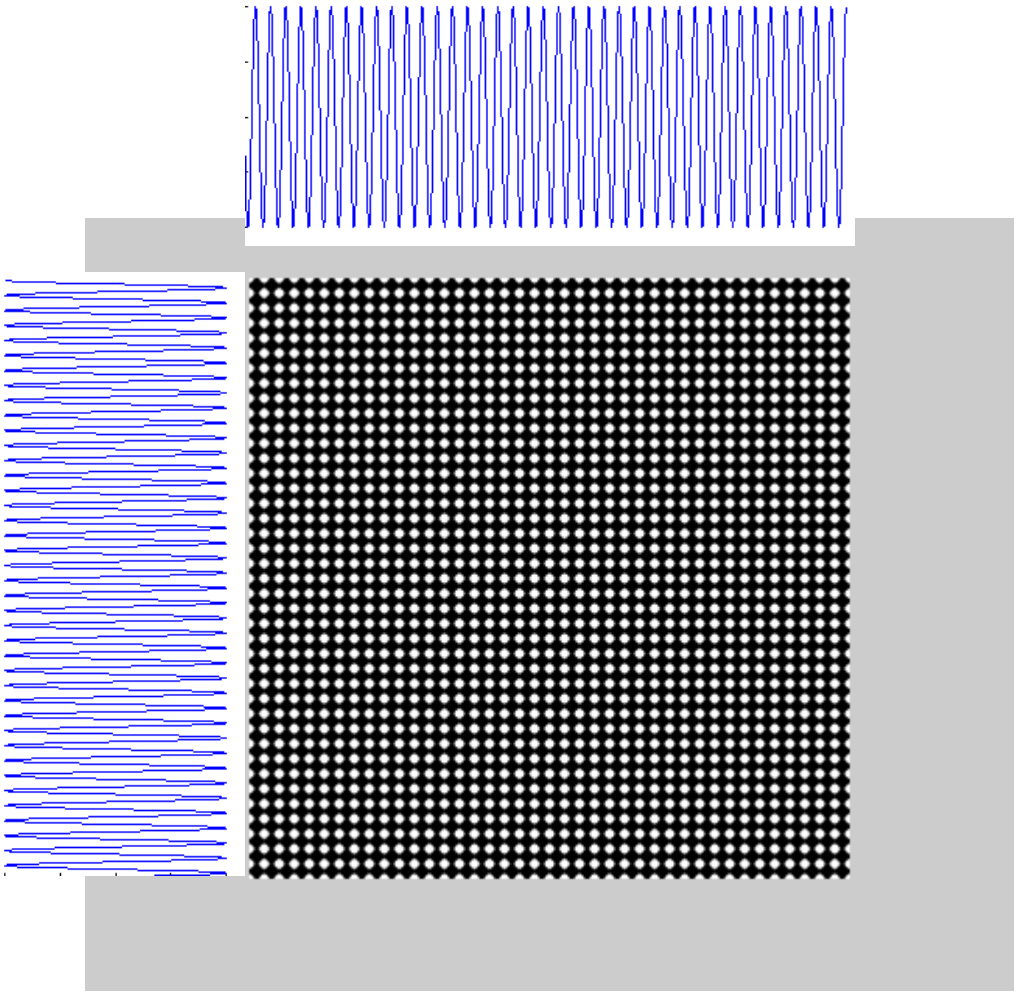


Frequency



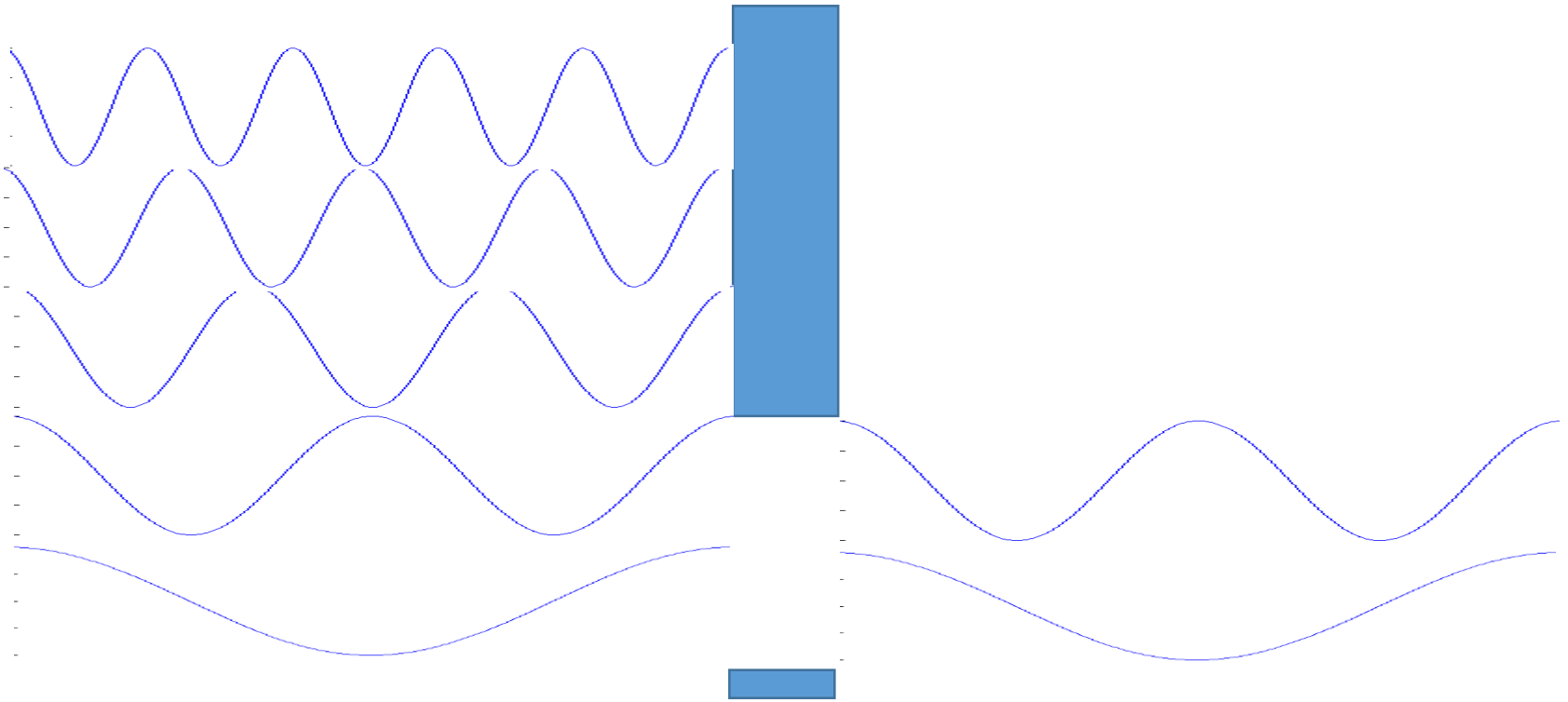
Frequency



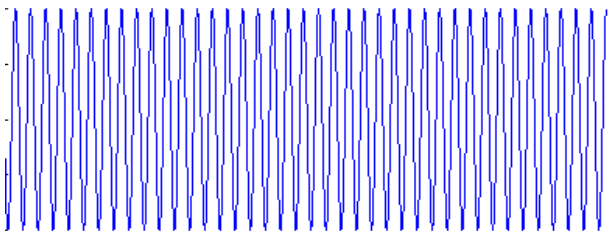


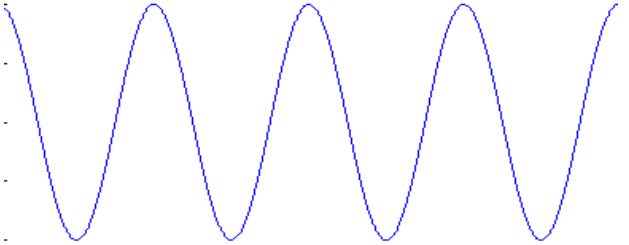
Frequency

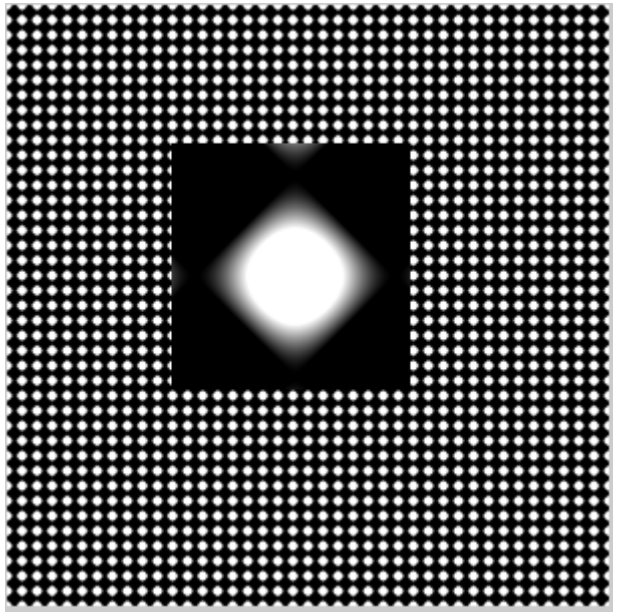
Low-pass filter

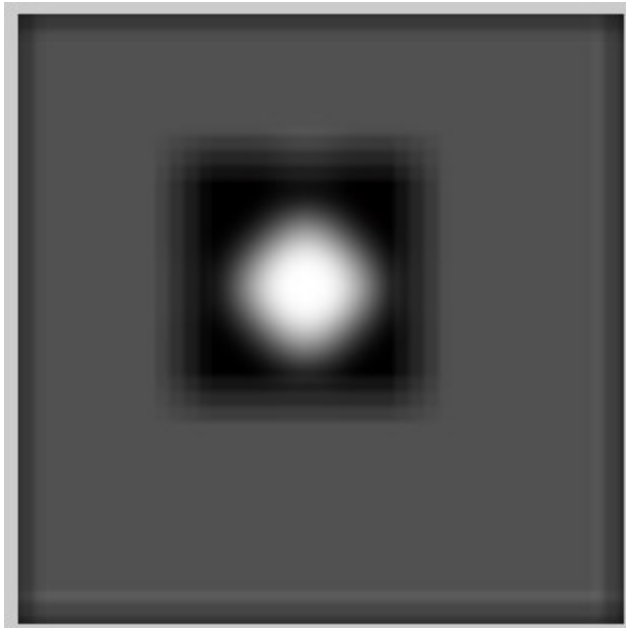


Low-pass filter

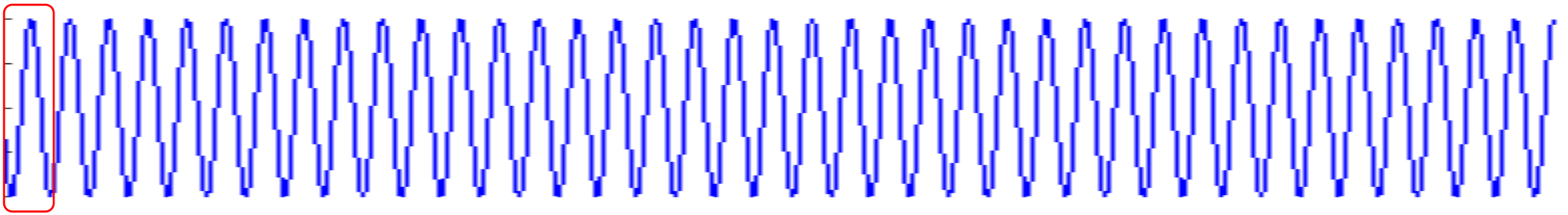




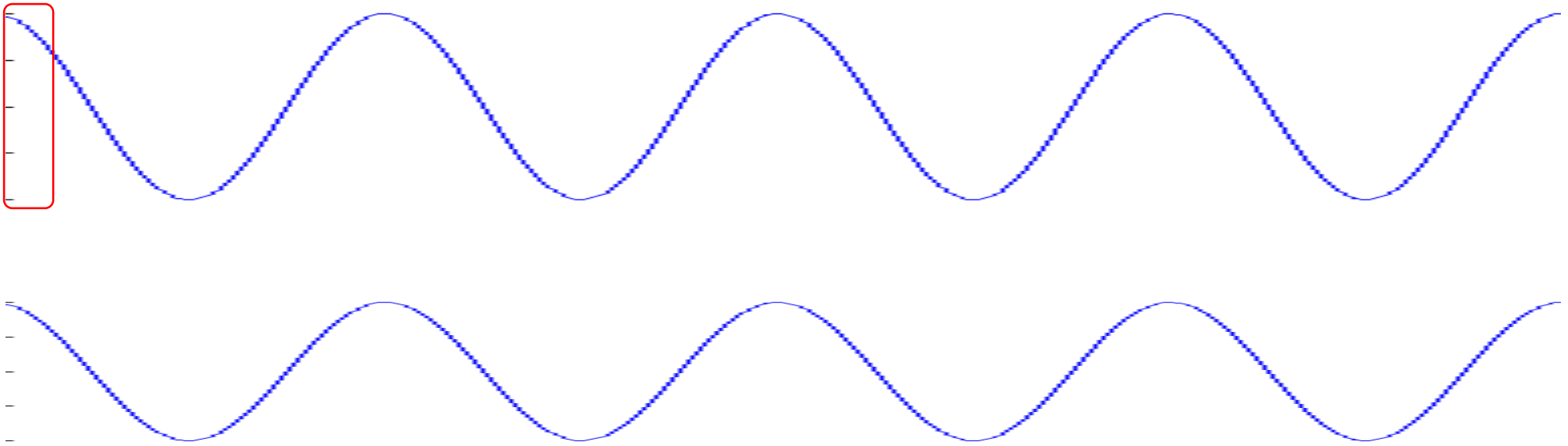




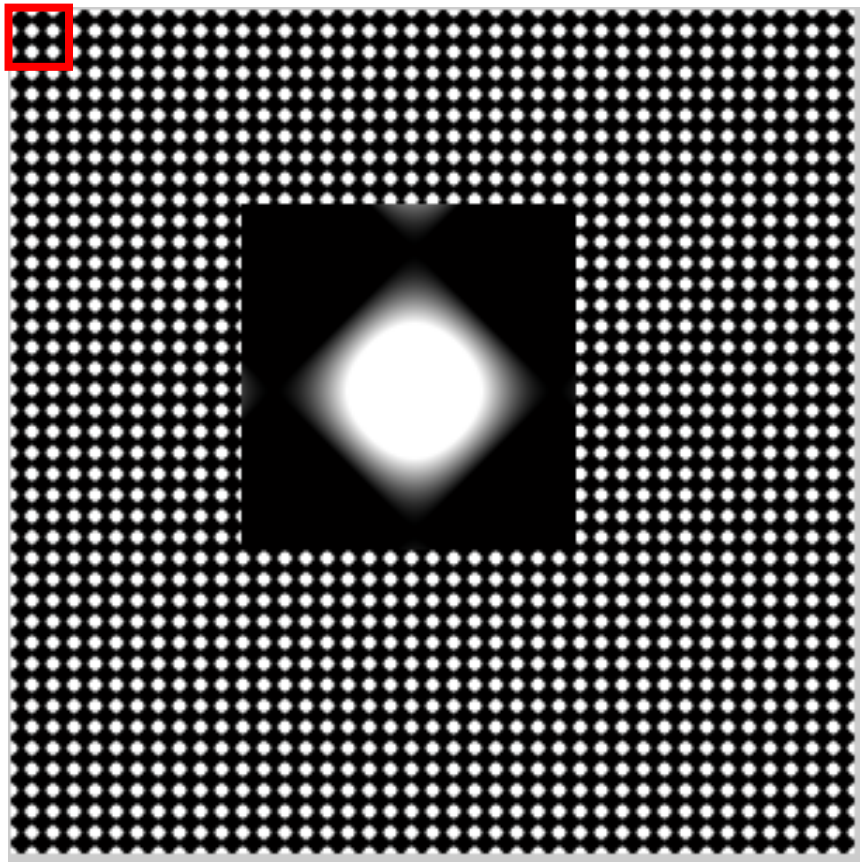
Low-pass filter



AVERAGE



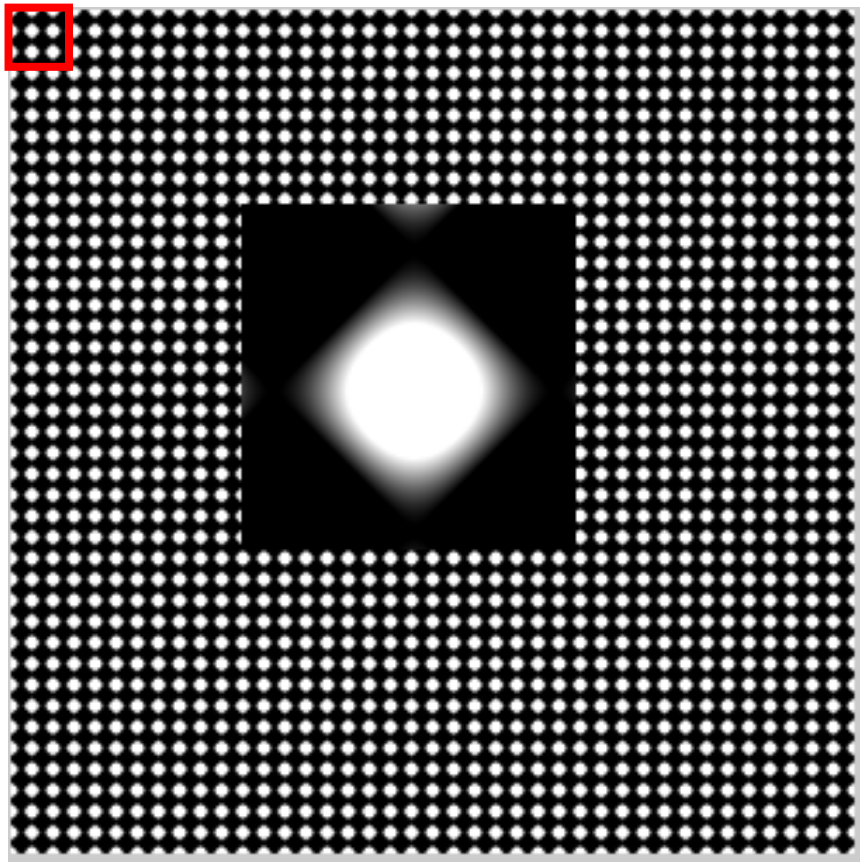
Low-pass filter

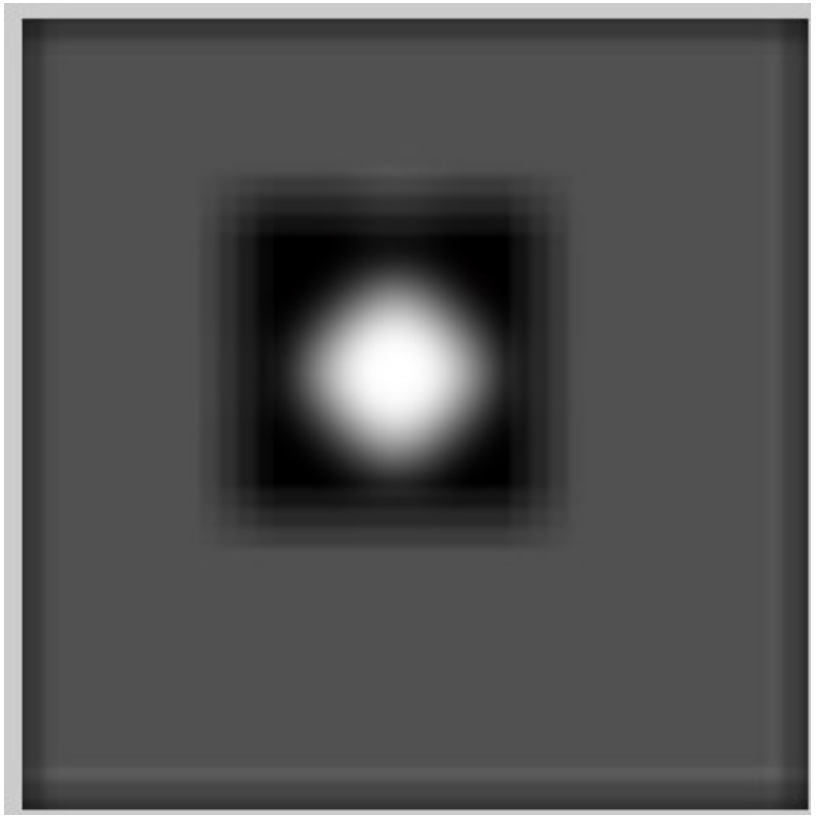


Arithmetic mean

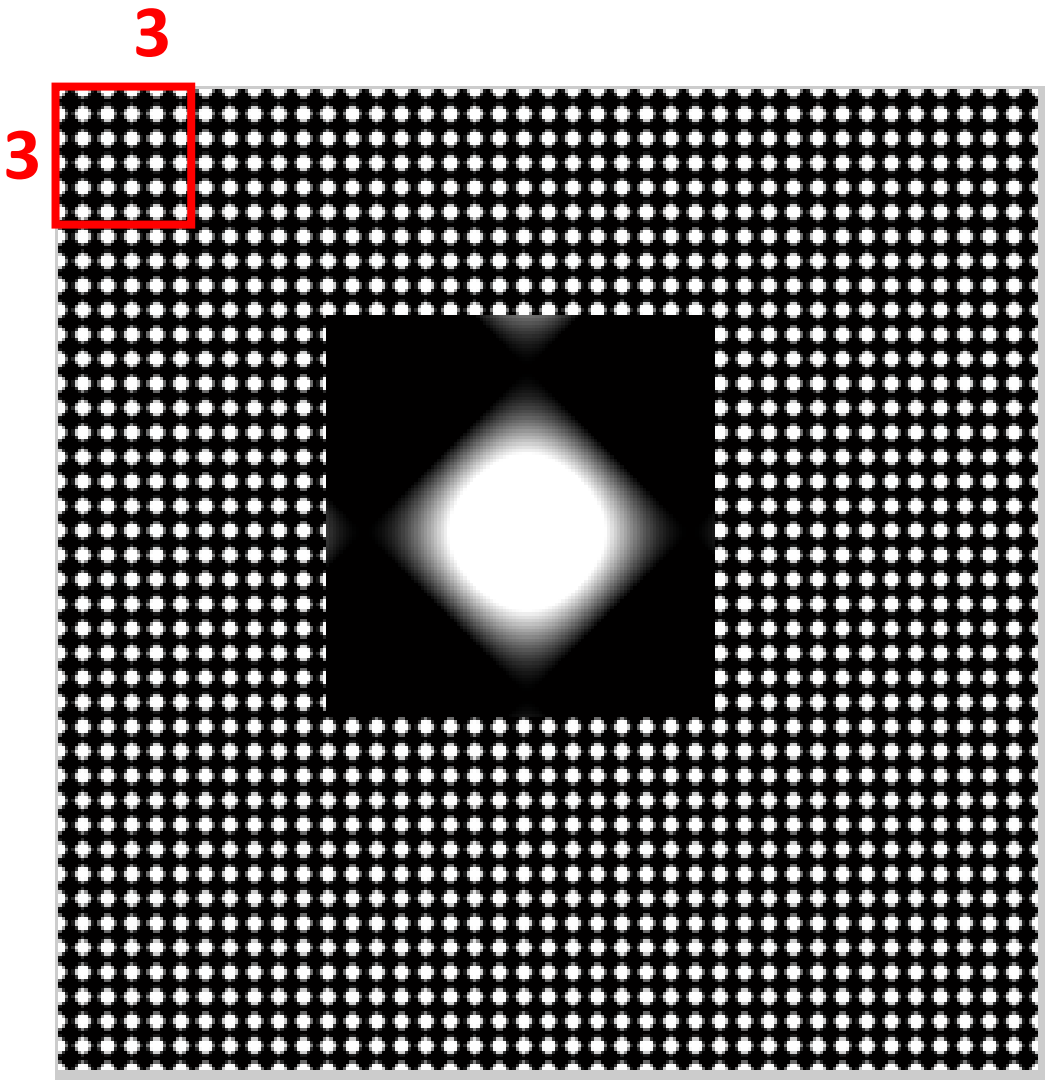
$$k = \sum_{X} \sum_{Y} \boldsymbol{I}(x, y) \,/ (|X| \times |Y|)$$

Low-pass filter





Low-pass filter



*

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

Average Kernel

Low-pass filter

Average Kernel

1/4	1/4
1/4	1/4

2x2

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

3x3

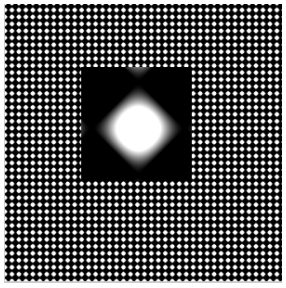
1/16	1/16	1/16	1/16
1/16	1/16	1/16	1/16
1/16	1/16	1/16	1/16
1/16	1/16	1/16	1/16

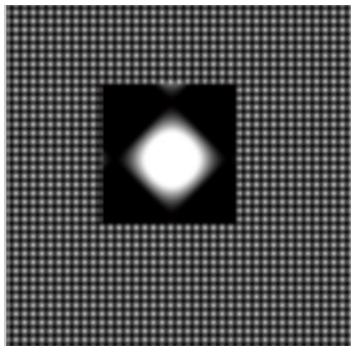
4x4

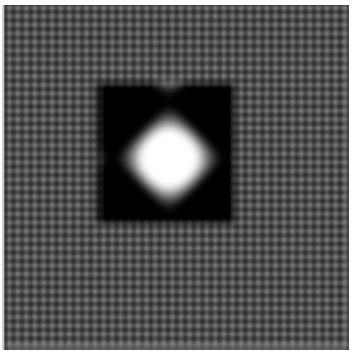
Low-pass filter

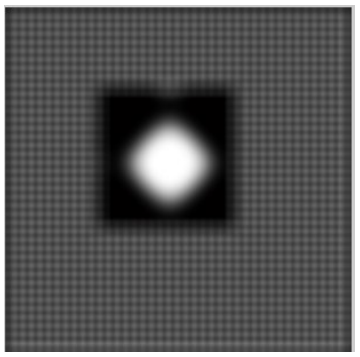
Average Kernel

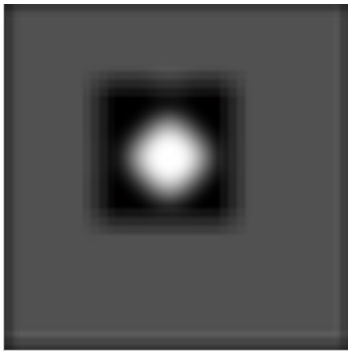
5x5 **10x10** **20x20** **30x30** **40x40**

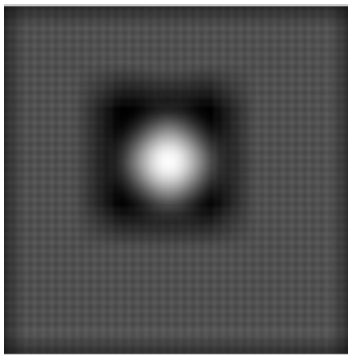












Low-pass filter

salt & pepper noise

2x2

3x3

4x4

5x5

speckle

2x2

3x3

4x4

5x5

Low-pass filter

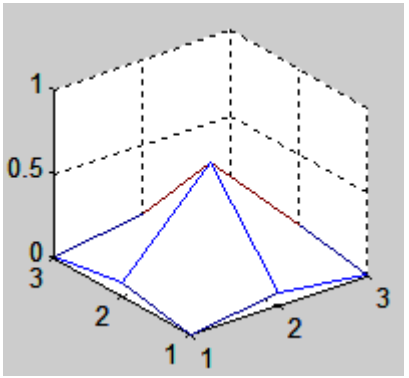
Gaussian Kernel

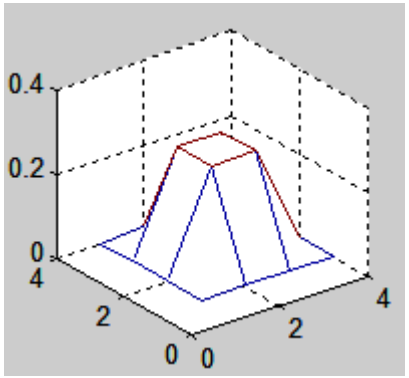
$$G(x,y) = \frac{1}{2\pi\sigma^e} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

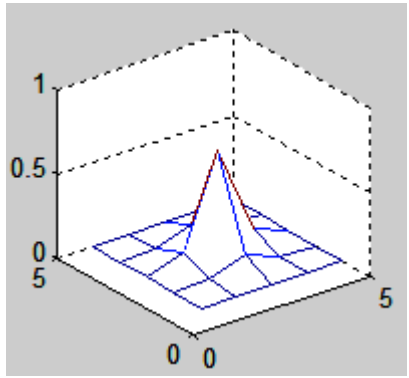
0.0113	0.0838	0.0113
0.0838	0.6193	0.0838
0.0113	0.0838	0.0113

0.0001	0.0044	0.0044	0.0001
0.0044	0.2411	0.2411	0.0044
0.0044	0.2411	0.2411	0.0044
0.0001	0.0044	0.0044	0.0001

0.0000	0.0000	0.0002	0.0000	0.0000
0.0000	0.0113	0.0837	0.0113	0.0000
0.0002	0.0837	0.6187	0.0837	0.0002
0.0000	0.0113	0.0837	0.0113	0.0000
0.0000	0.0000	0.0002	0.0000	0.0000





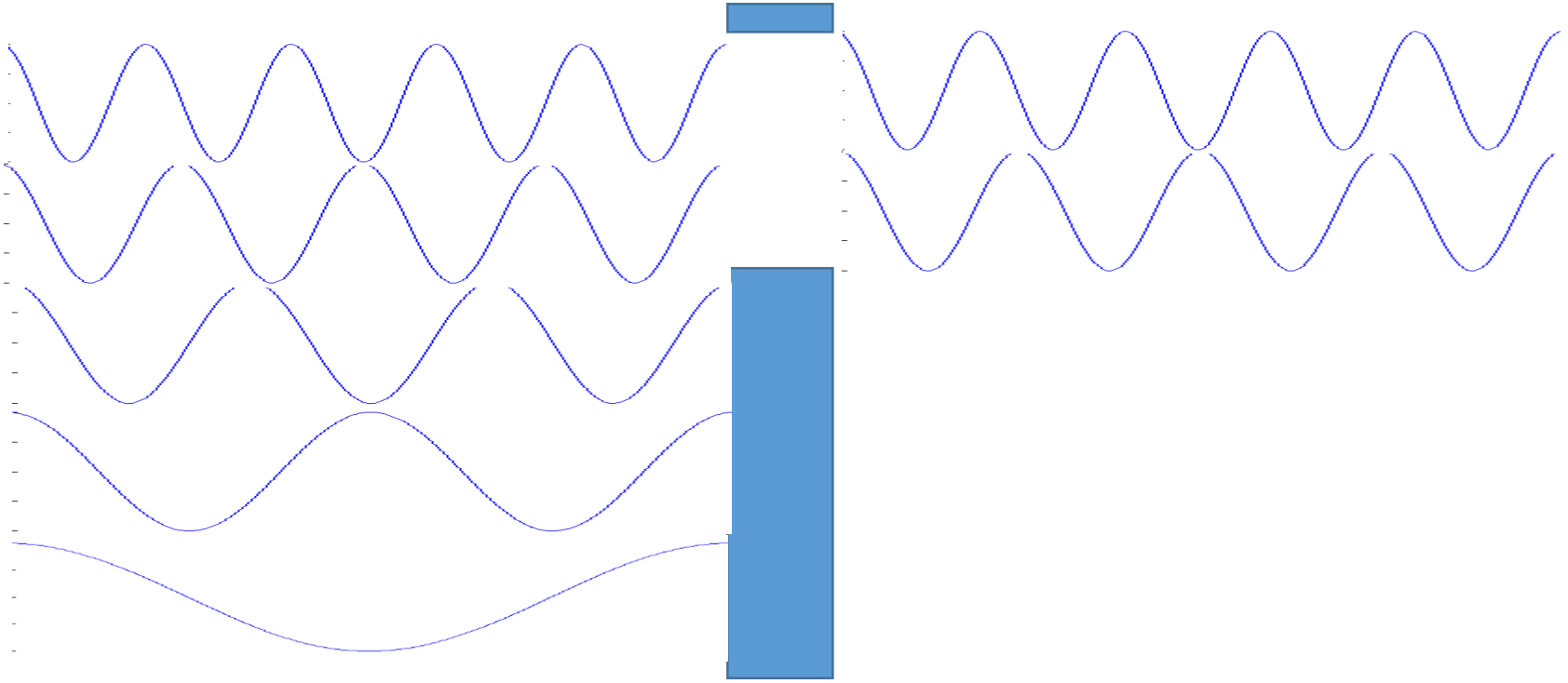


Low-pass filter

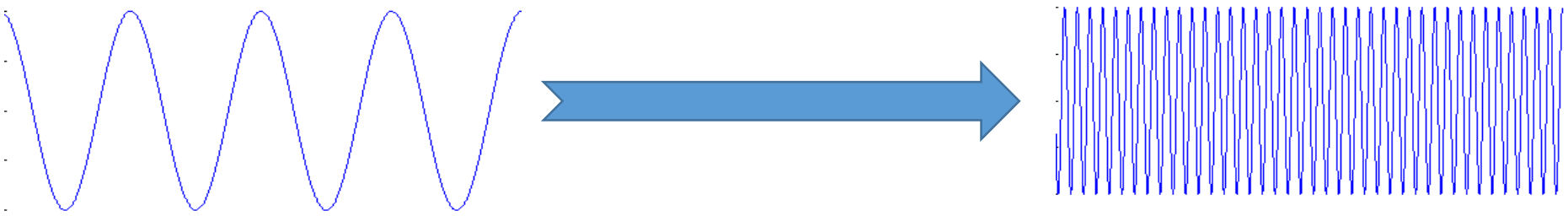
Gaussian 5x5, σ=1.2

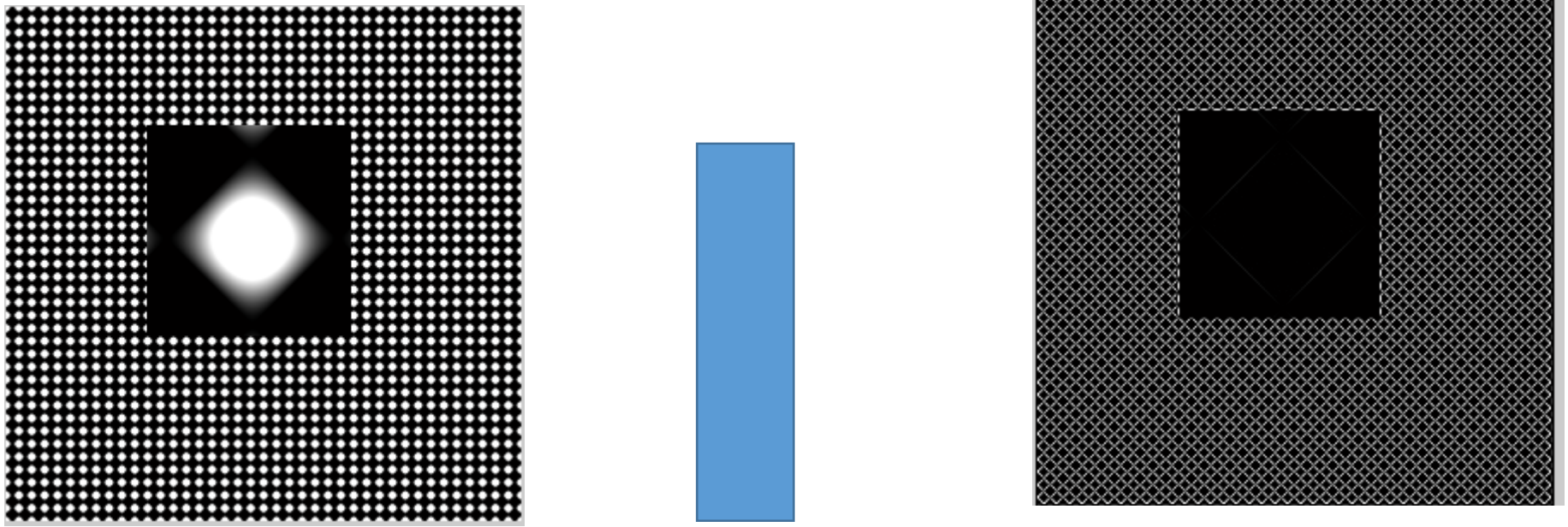
Average 5x5

High-pass filter



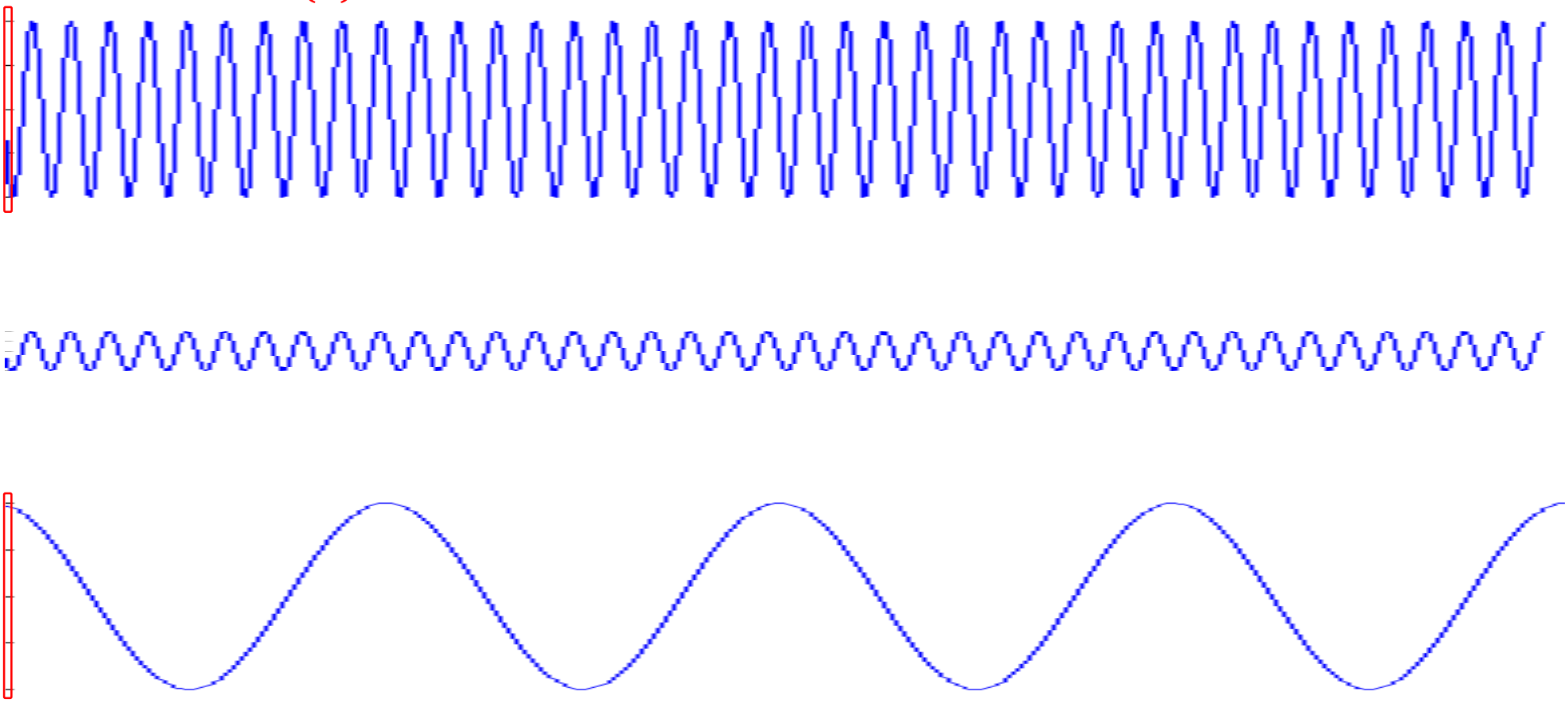
High-pass filter



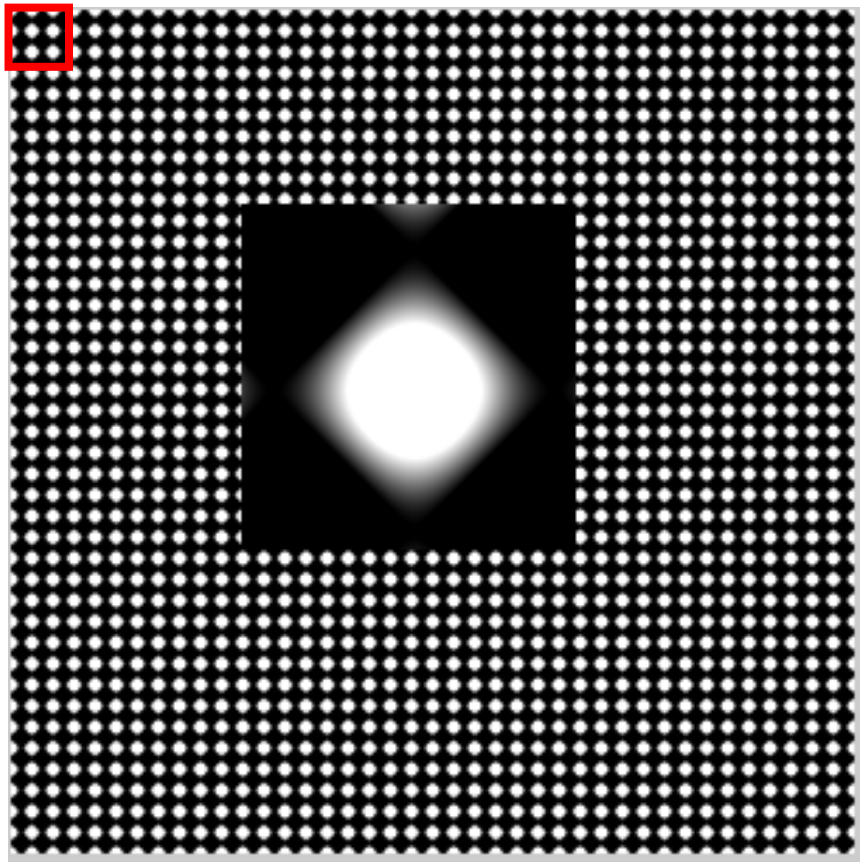


High-pass filter

Derivative $\frac{\partial(y)}{\partial(x)}$



High-pass filter



2D gradient

$$k = \frac{\partial I}{\partial x} + \frac{\partial I}{\partial y}$$

High-pass filter

Laplacian Kernel

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\nabla^2 = \frac{4}{\alpha + 1}\begin{bmatrix} \frac{\alpha}{4} & \frac{1-\alpha}{4} & \frac{\alpha}{4} \\ \frac{1-\alpha}{4} & -1 & \frac{1-\alpha}{4} \\ \frac{\alpha}{4} & \frac{1-\alpha}{4} & \frac{\alpha}{4} \end{bmatrix}$$

$$\mathbf{0 \leq \alpha \leq 1}$$

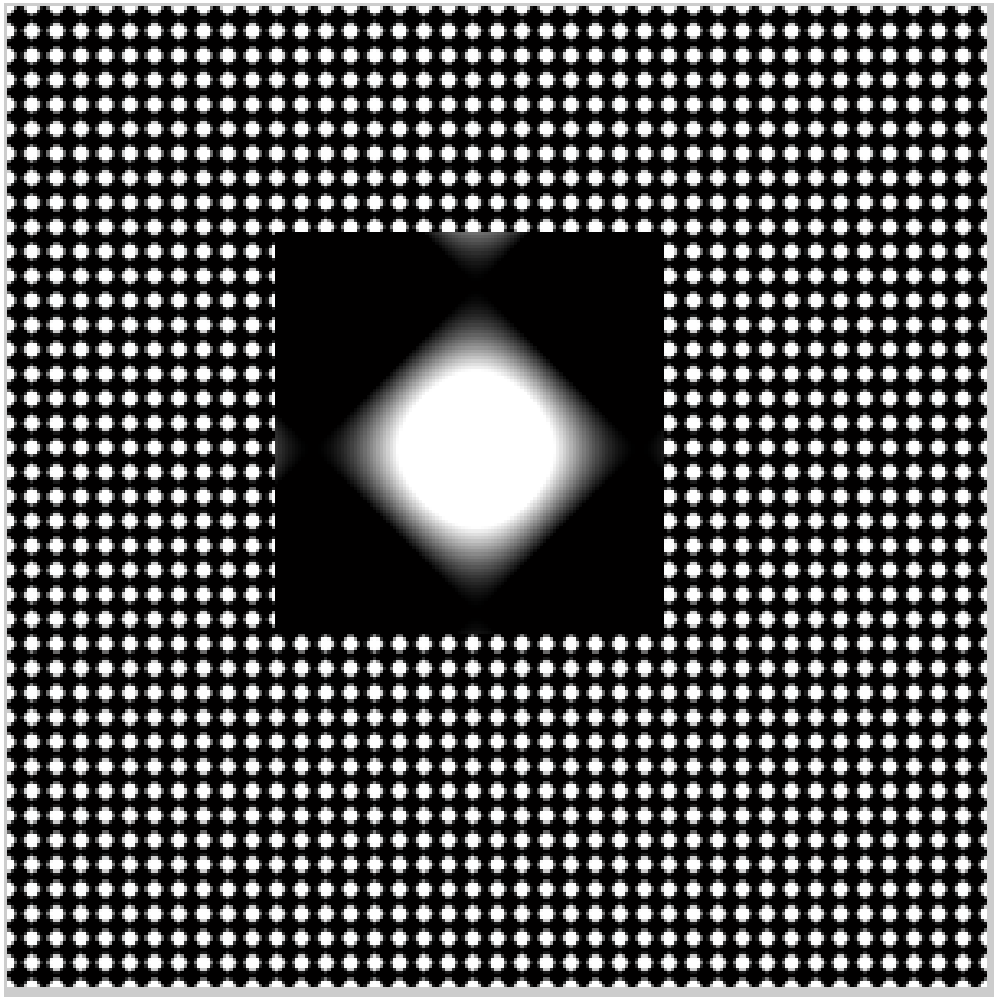
High-pass filter

Laplacian Kernel

$$\boldsymbol{\alpha = 0}, \quad \nabla^2 = \begin{matrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{matrix}$$

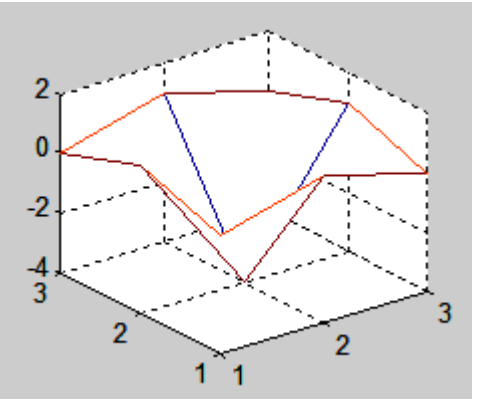
$$\boldsymbol{\alpha = 1}, \quad \nabla^2 = \begin{matrix} .5 & 0 & .5 \\ 0 & -2 & 0 \\ .5 & 0 & .5 \end{matrix}$$

High-pass filter

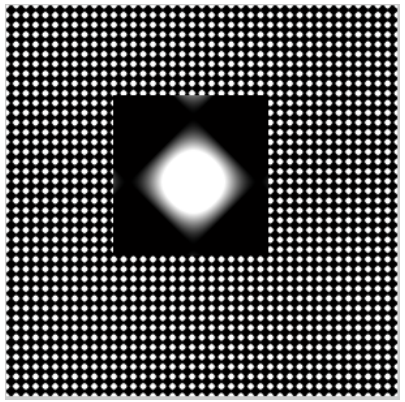


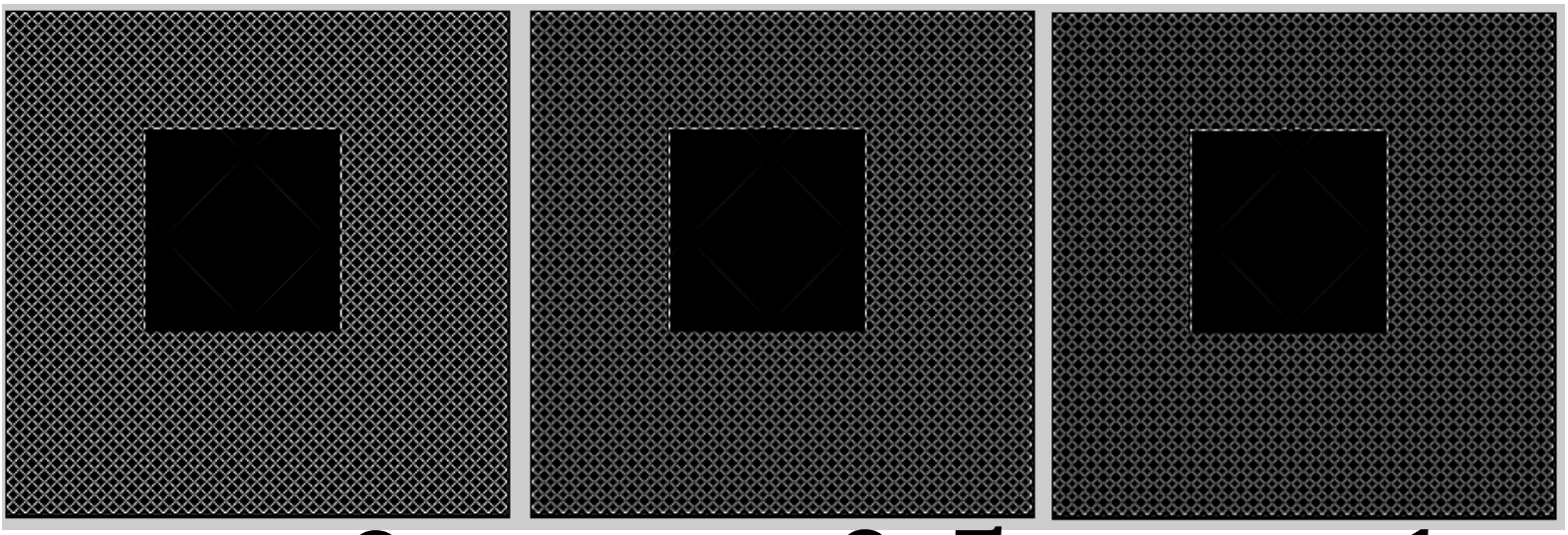
*

0	1	0
1	-4	1
0	1	0

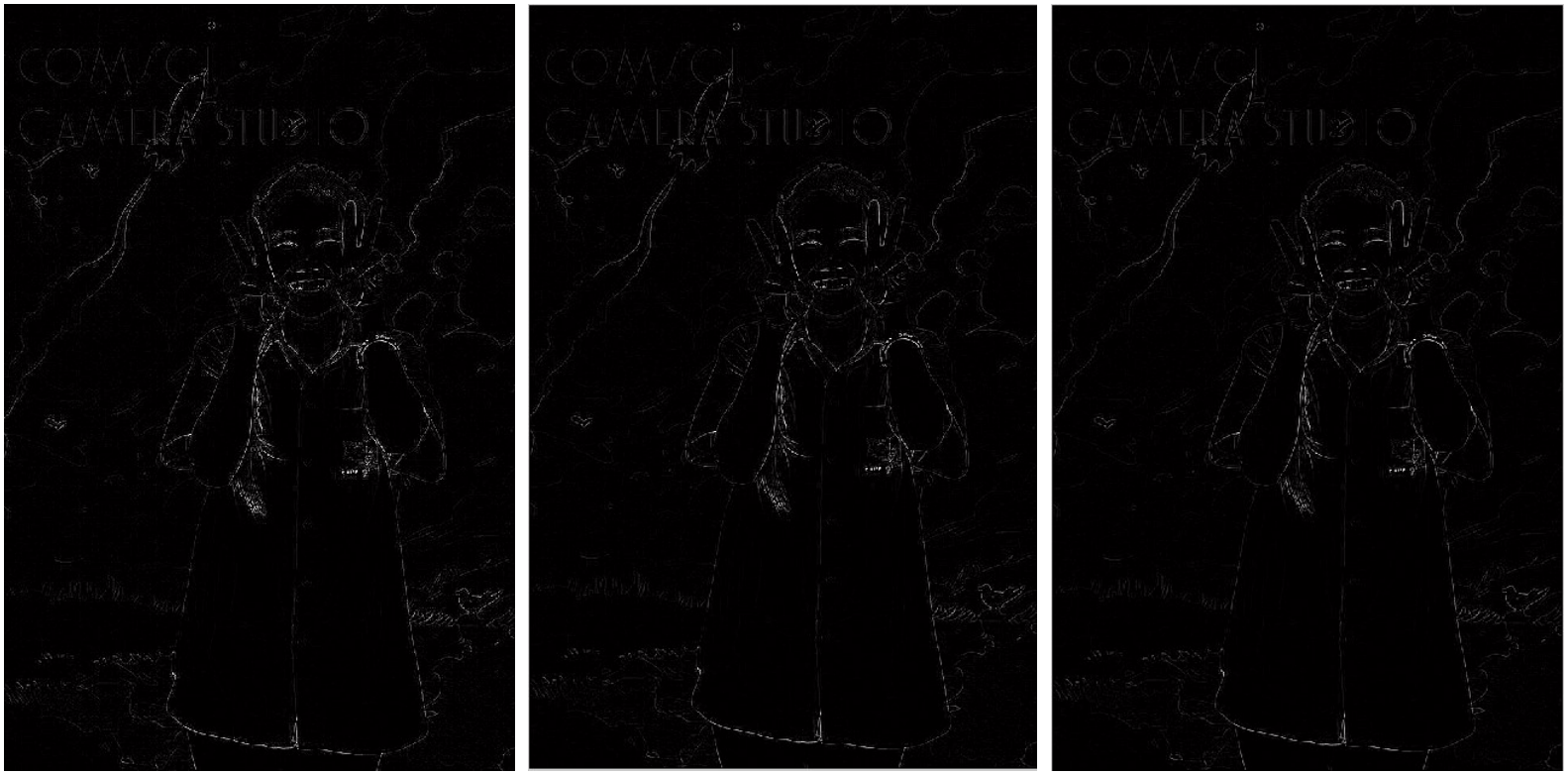


High-pass filter





$\alpha = 0$ $\alpha = 0.5$ $\alpha = 1$



High-pass filter

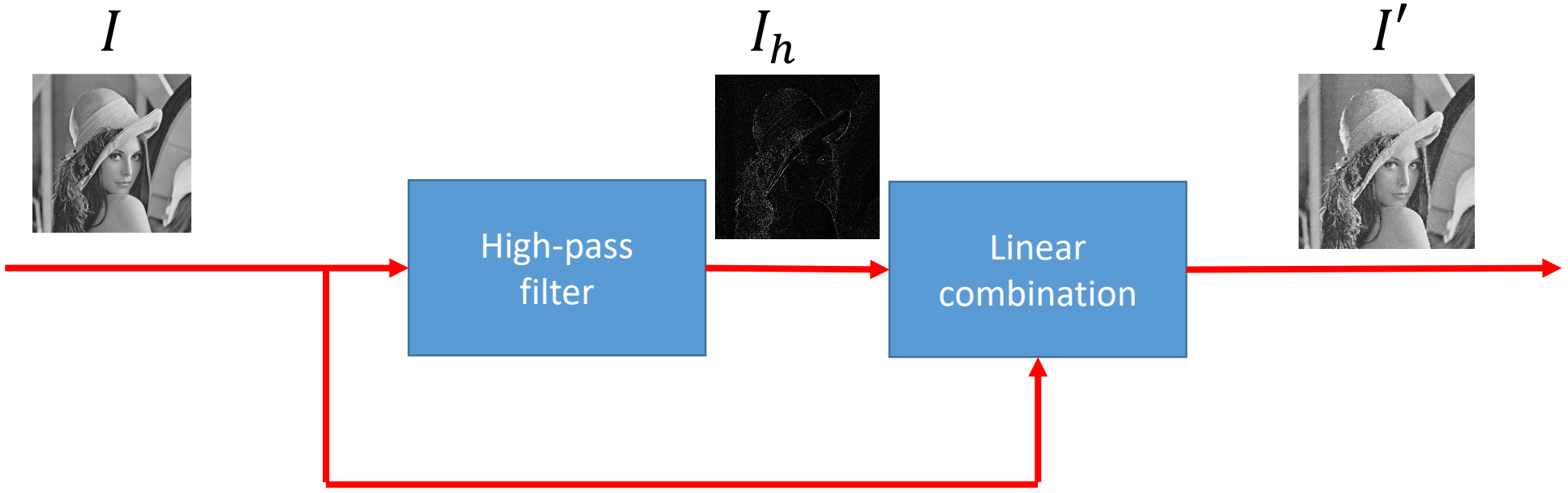


Image Sharpening Algorithm

$$I' = I + wI_h$$

High-pass filter

$\alpha = 0, w = 0.4$

$\alpha = 0, w = 0.6$

$\alpha = 0, w = 10$

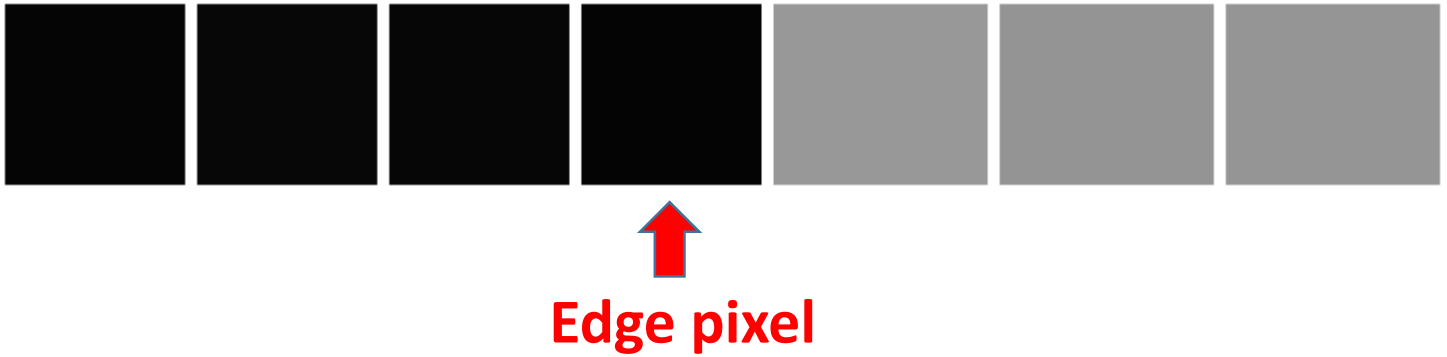
High-pass filter

$$\begin{matrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{matrix}$$

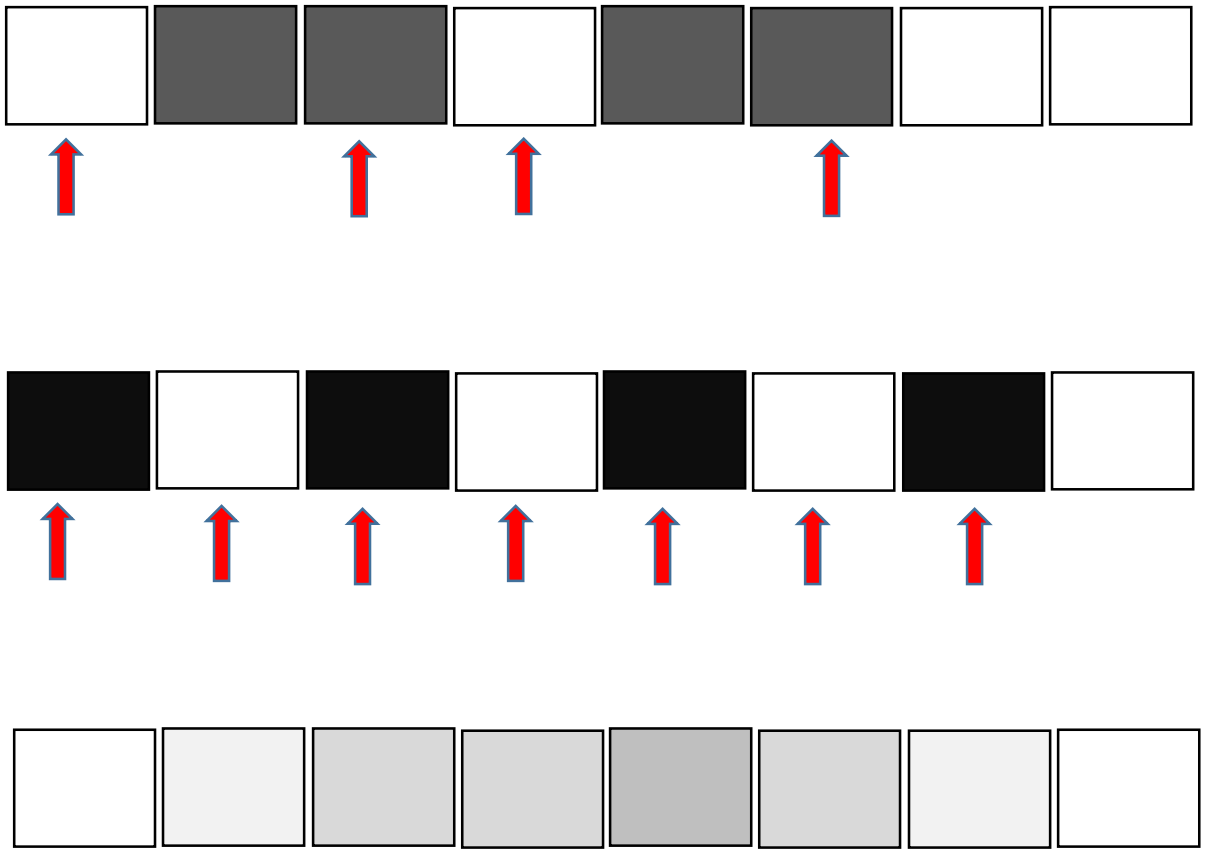
$$w = 0.4$$

Edge detection

Edge detection is the name for a set of mathematical methods which aim at identifying **points** in a digital image at which the image brightness changes sharply or, more formally, has **discontinuities**.



Edge detection

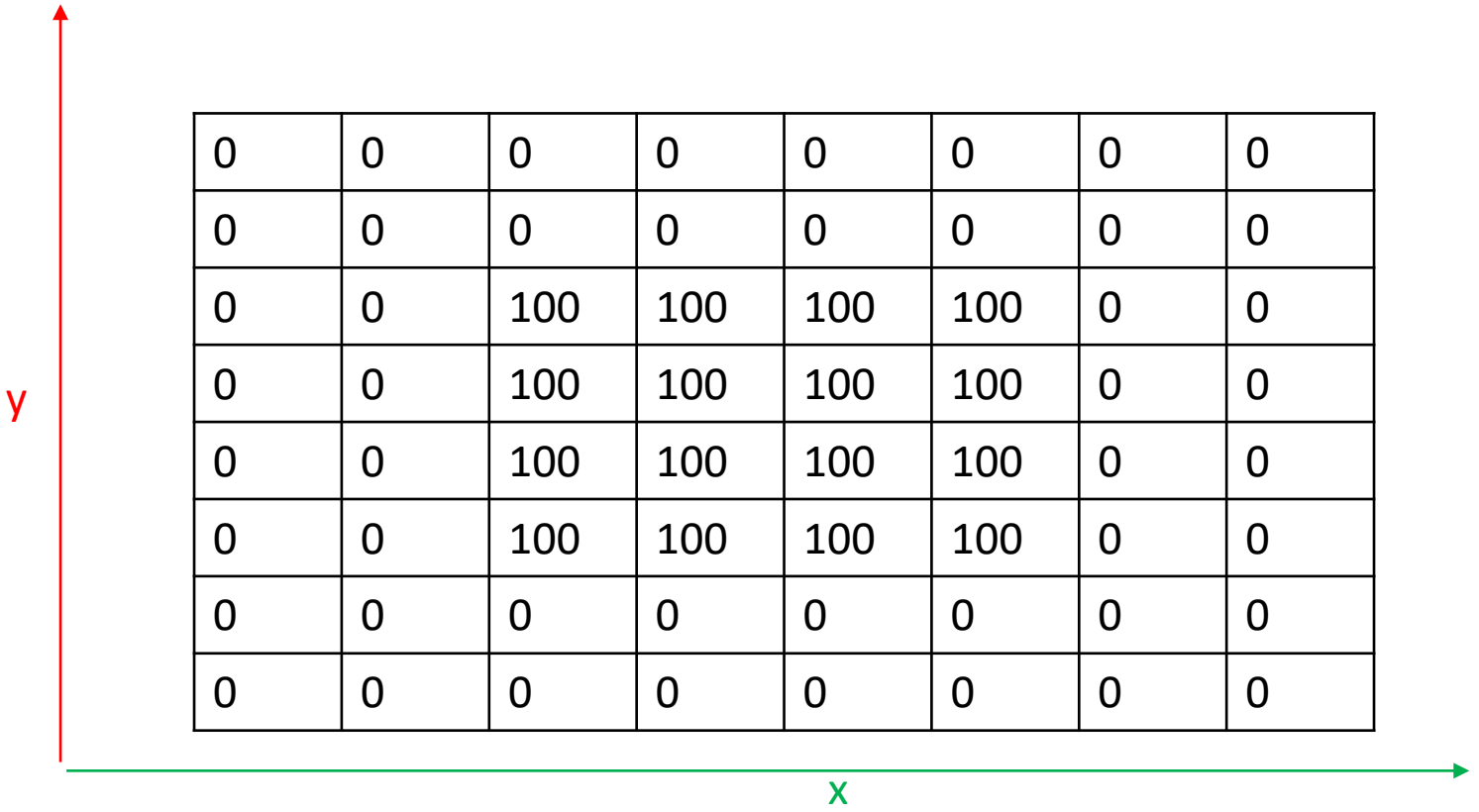


$$Edge_i = I_{i-1} - I_i$$

Edge detection

Gradient Magnitude Edge Detection Method

Edge detection(Gradient Magnitude)



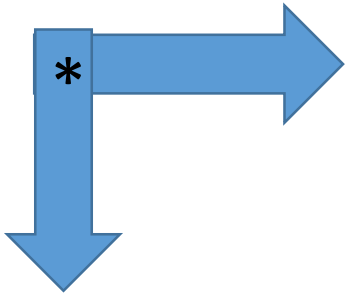
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Edge detection(Gradient Magnitude)

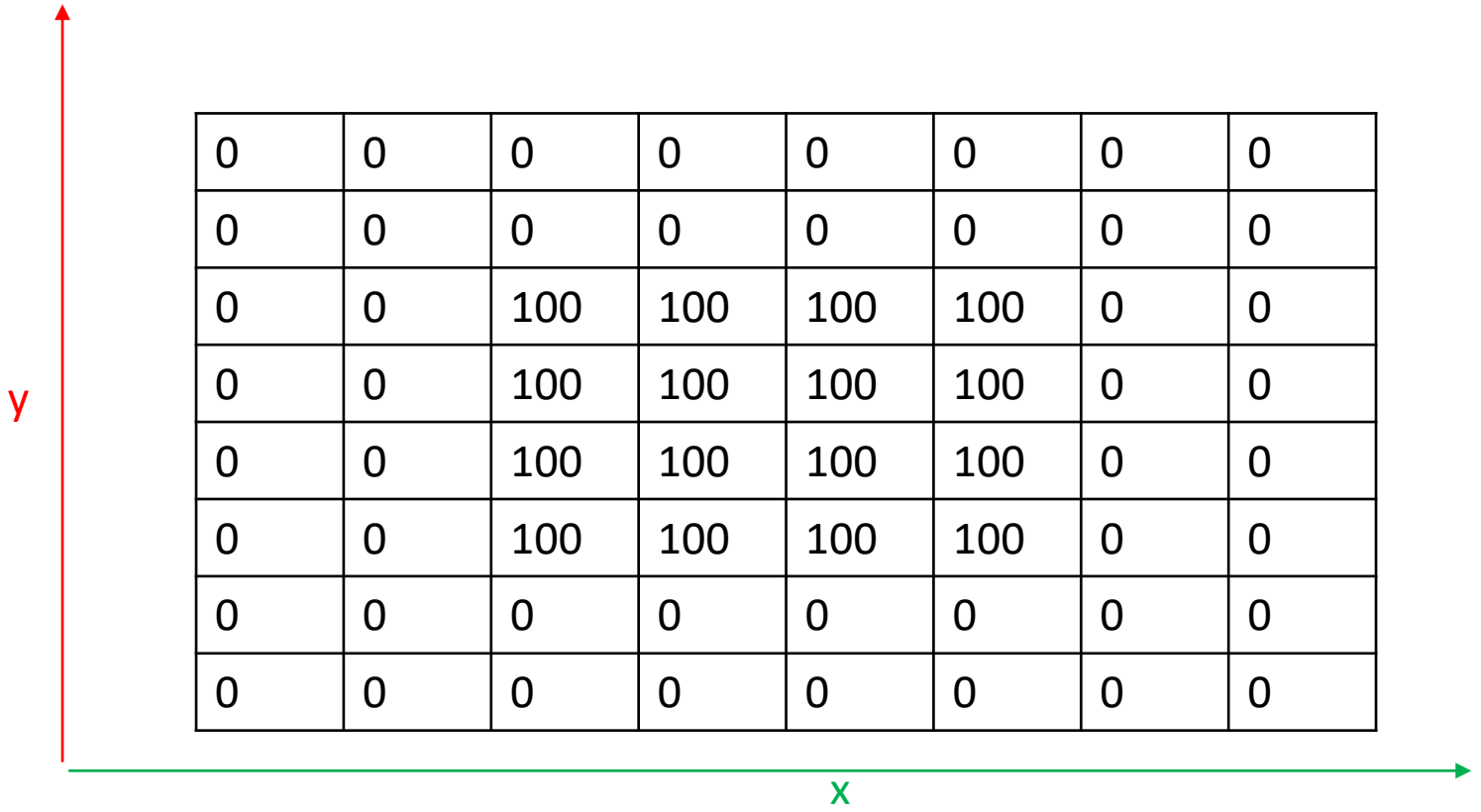
y

0	0	0	0	0	0	0	0
0	0	**0**	**0**	**0**	**0**	0	0
0	**0**	100	100	100	**100**	0	0
0	**0**	100	100	100	**100**	0	0
0	**0**	100	100	100	**100**	0	0
0	0	100	100	100	100	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

x



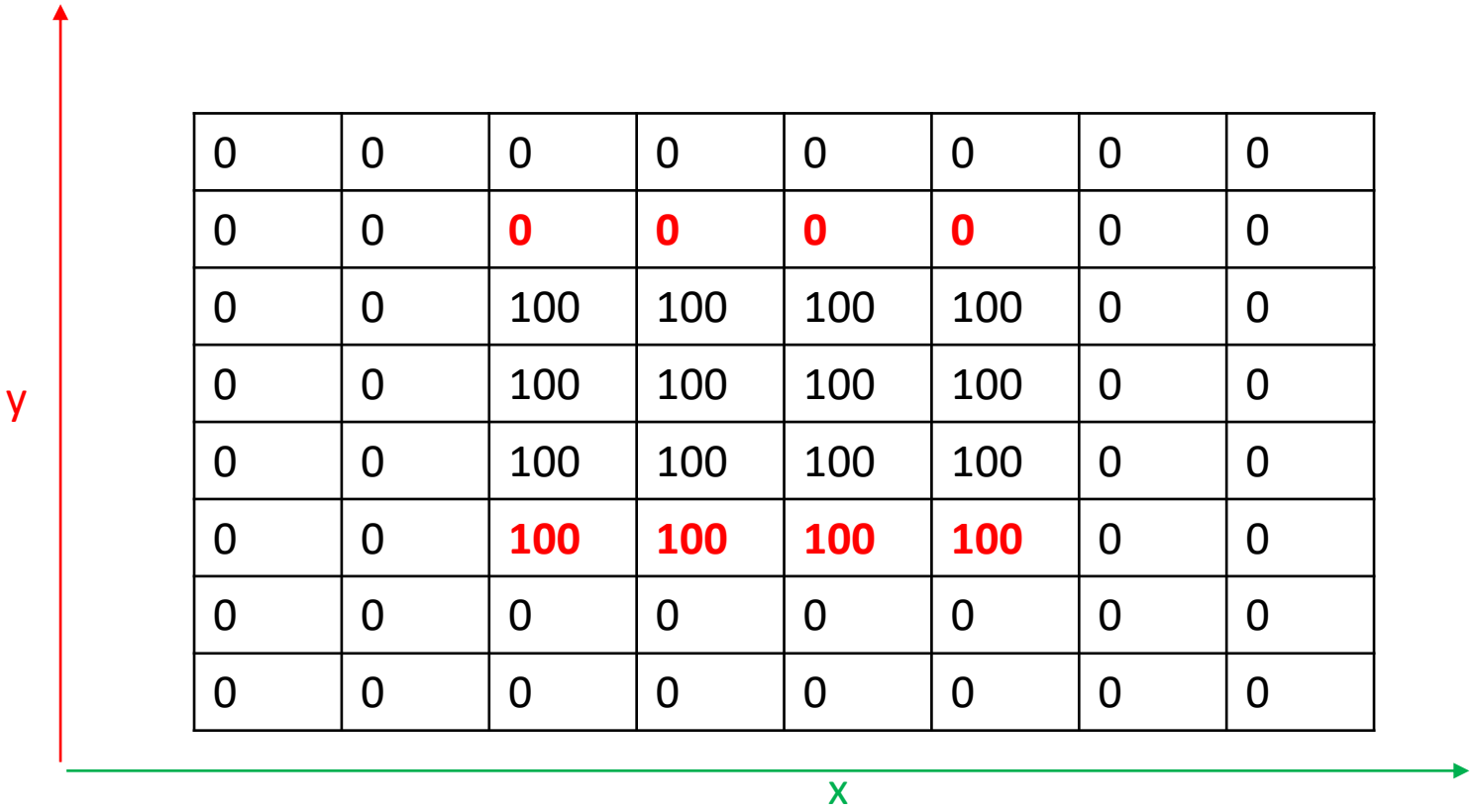
Edge detection(Gradient Magnitude)



0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

∇y Horizontal Edge

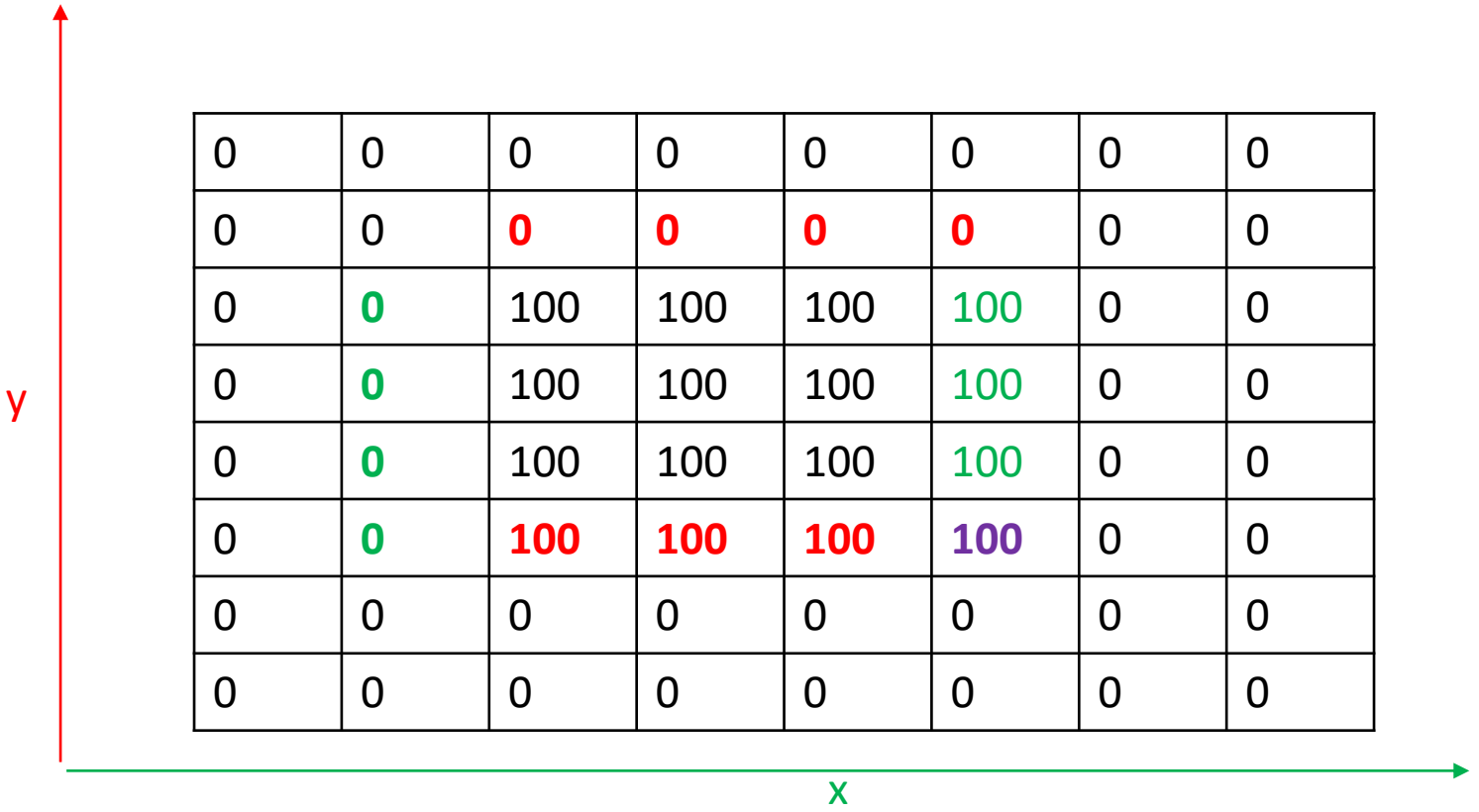
Edge detection(Gradient Magnitude)



0	0	0	0	0	0	0	0
0	0	**0**	**0**	**0**	**0**	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	**100**	**100**	**100**	**100**	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

∇y Horizontal Edge

Edge detection(Gradient Magnitude)

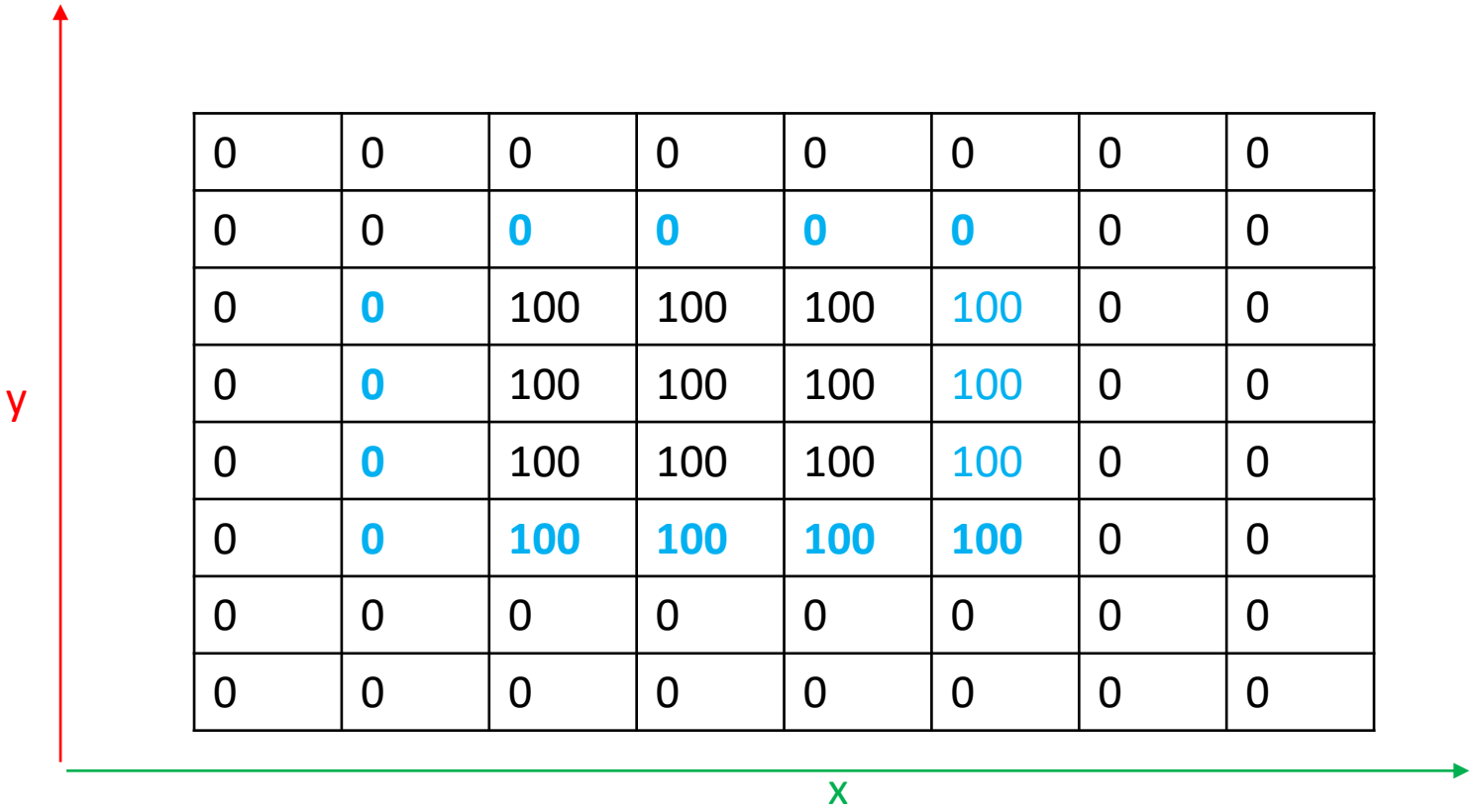


0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

∇y Horizontal Edge

∇x Vertical Edge

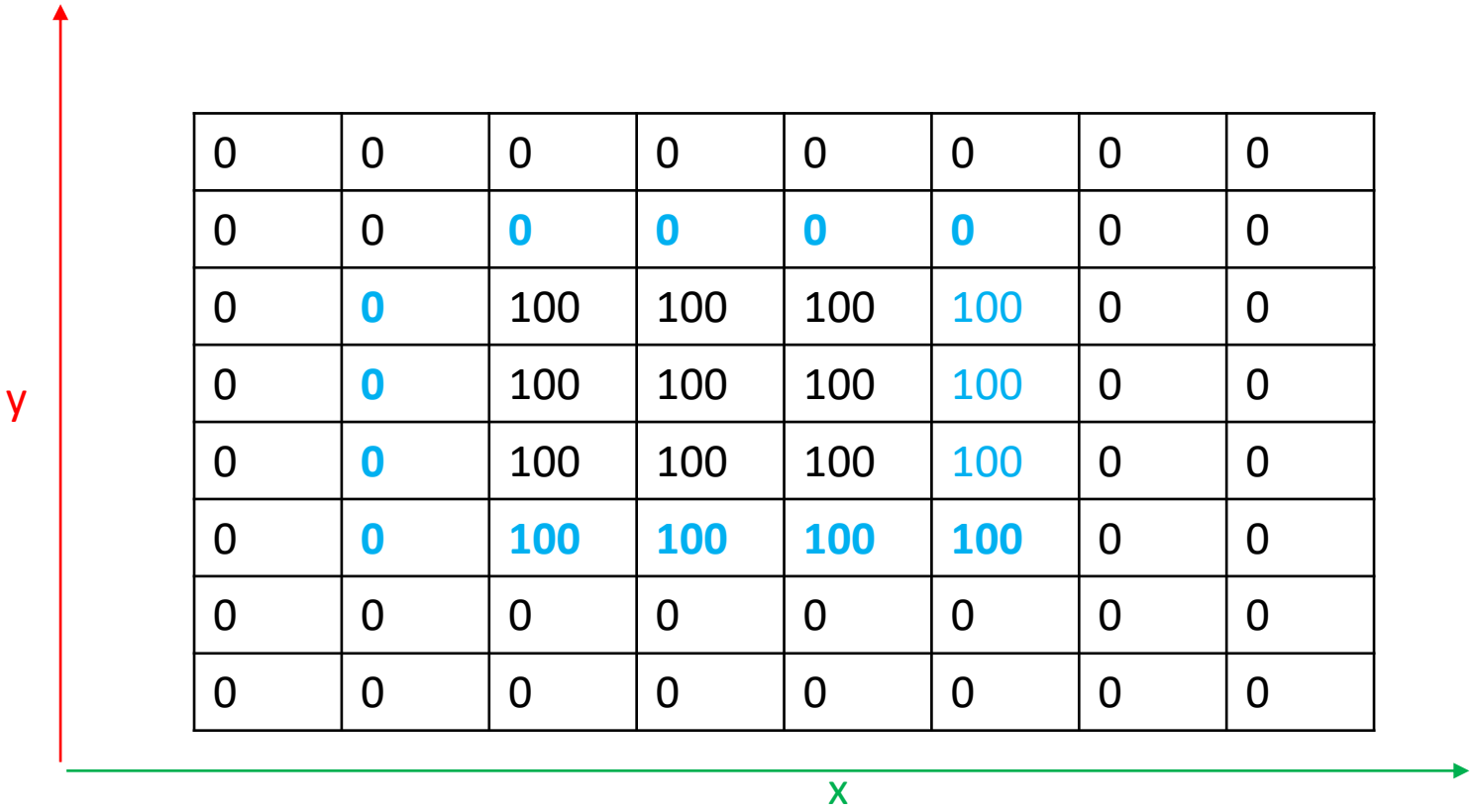
Edge detection(Gradient Magnitude)



0	0	0	0	0	0	0	0
0	0	**0**	**0**	**0**	**0**	0	0
0	**0**	100	100	100	100	0	0
0	**0**	100	100	100	100	0	0
0	**0**	100	100	100	100	0	0
0	**0**	**100**	**100**	**100**	**100**	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$$\nabla y + \nabla x$$

Edge detection(Gradient Magnitude)



0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$$\nabla y + \nabla x$$

-1
1

-1	1

Edge detection(Gradient Magnitude)

I

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$I * [-1 \quad 1]$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	-100	0	0	0	100	0
0	-100	0	0	0	100	0
0	-100	0	0	0	100	0
0	-100	0	0	0	100	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Edge detection(Gradient Magnitude)

I

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$I * \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

0	0	0	0	0	0	0	0
0	0	-100	-100	-100	-100	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	100	100	100	100	0	0
0	0	0	0	0	0	0	0

Edge detection(Gradient Magnitude)

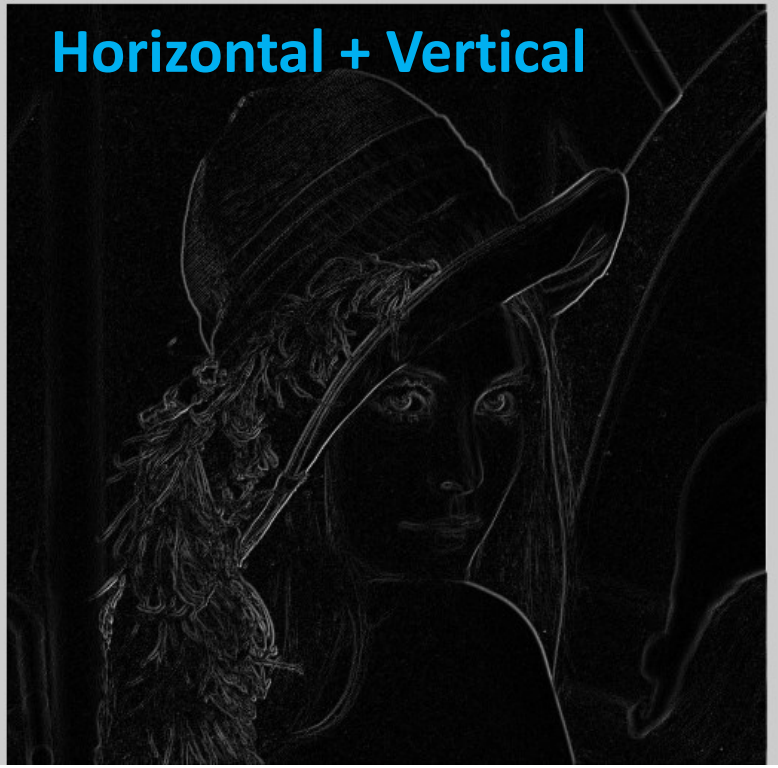
I

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	100	100	100	100	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$$abs(I * \begin{bmatrix} -1 \\ 1 \end{bmatrix}) + abs(I * [-1 \quad 1])$$

0	0	0	0	0	0	0	0
0	0	100	100	100	100	0	0
0	100	0	0	0	100	0	0
0	100	0	0	0	100	0	0
0	100	0	0	0	100	0	0
0	100	100	100	100	200	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Edge detection(Gradient Magnitude)



Edge detection(Prewitt)

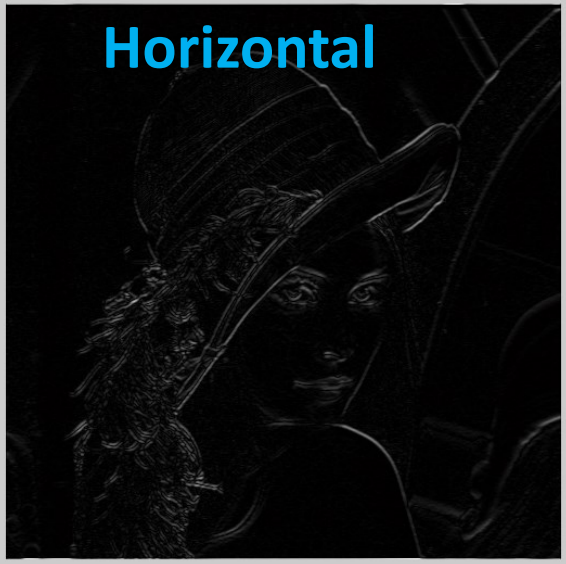
1	**1**	**1**
0	**0**	**0**
-1	**-1**	**-1**

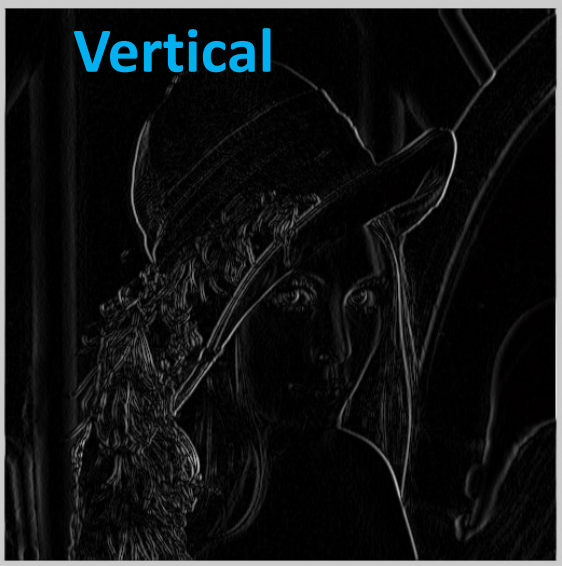
Prewitt Horizontal

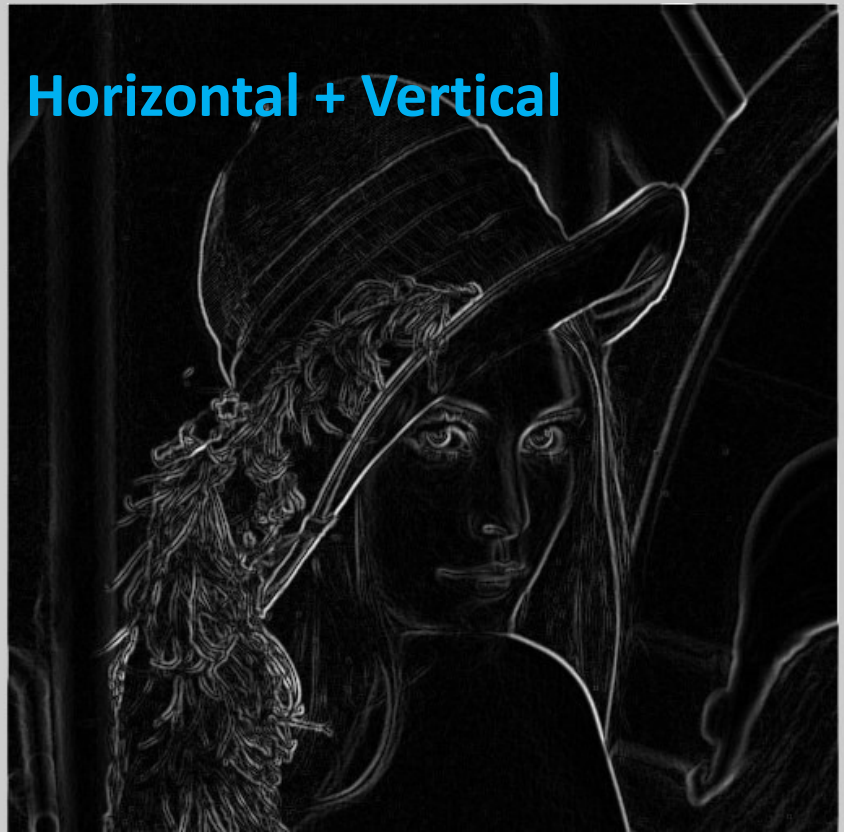
1	**0**	**-1**
1	**0**	**-1**
1	**0**	**-1**

Prewitt Vertical

Edge detection(Prewitt)







Edge detection(Sobel)

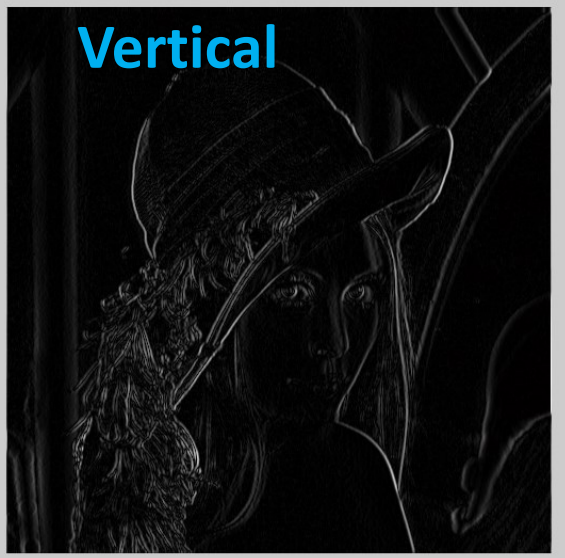
1	**2**	**1**
0	**0**	**0**
-1	**-2**	**-1**

Sobel Horizontal

1	**0**	**-1**
2	**0**	**-2**
1	**0**	**-1**

Sobel Vertical

Edge detection(Sobel)



Edge detection(Robert)

0	**1**
-1	**0**

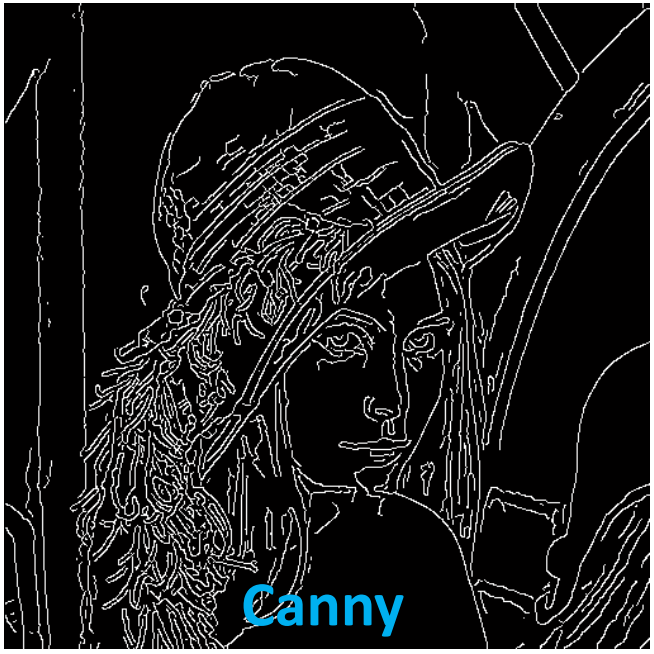
Robert Horizontal

1	**0**
0	**-1**

Robert Vertical

Edge detection(Robert)

Edge detection



Thresholding

Thresholding is the simplest method of image segmentation. From a grayscale image, thresholding can be used to create binary images

$$I_{binary}(j) = \begin{cases} 1 \; where \; I(j) \geq \theta \\ 0 \; where \; I(j) < \theta \end{cases}$$

Thresholding

Thresholding

Original Image

Binary Image

$\theta = 0.1$

$\theta = 0.2$

$\theta = 0.3$

Question !

164	**239**	**53**	**50**	**79**
97	**223**	**77**	**58**	**235**
207	**140**	**120**	**44**	**110**
136	**159**	**59**	**58**	**47**
89	**150**	**215**	**111**	**231**

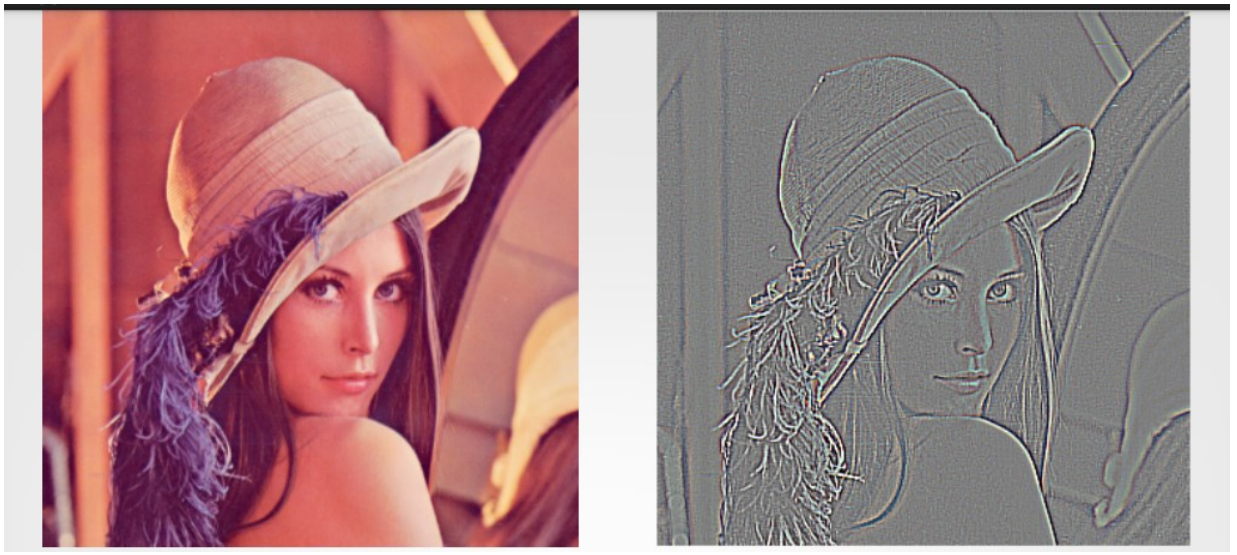
$\boldsymbol{I}$

Please convert uint8 image **I** to binary image using threshold

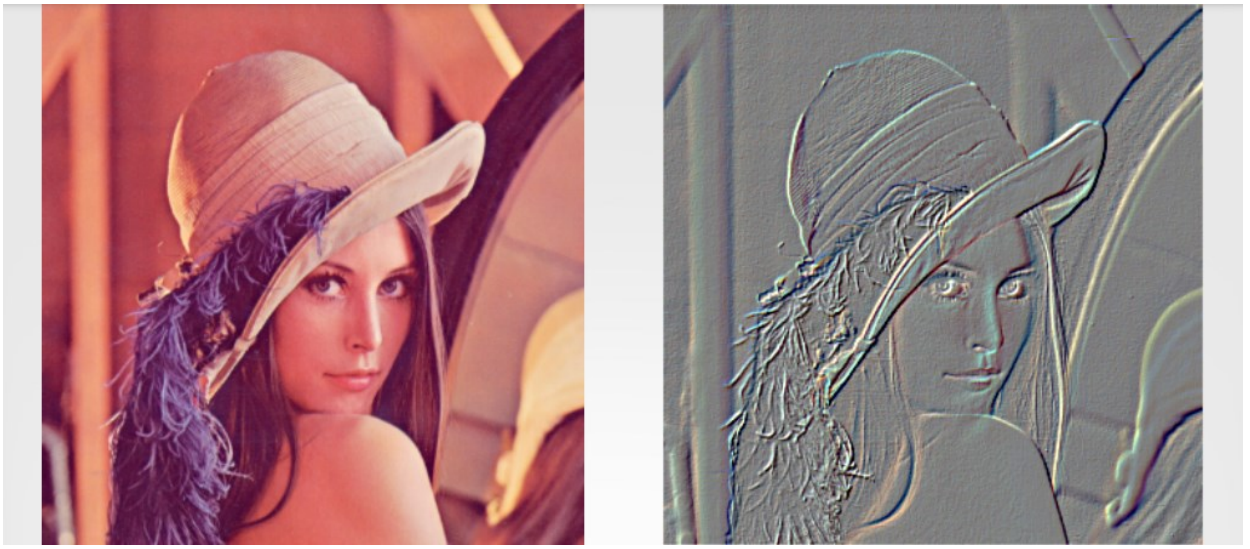
$\boldsymbol{\theta = 127}$

Sol..

1	**1**	**0**	**0**	**0**
0	**1**	**0**	**0**	**1**
1	**1**	**0**	**0**	**0**
1	**1**	**0**	**0**	**0**
0	**1**	**1**	**0**	**1**



Emboss



Engrave

Matlab image processing function

Kernel Generator:

H=fspecial('kernel type',param1,param2);

2D Convolution:

Im=conv2(**im,H**);

Image filter:

Im=imfilter(**im,H**);

Threshold:

Im=im2bw(**im**,*threshold*);

Image filter kernel

https://en.wikipedia.org/wiki/Kernel_(image_processing)